Cuaderno de notas de trabajo

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Cuaderno 44

Principio Variacional.

La variación de la integral J es nula:

$$J = \int_a^b \frac{m}{2} e^{\frac{2M}{(\hat{r}\cdot\hat{V})}} \left[2\hat{V} - \frac{\hat{V}}{(\hat{V}\cdot\hat{V})} \right] \bullet d\hat{\varrho};$$

$$\delta J = 0;$$

m —— masa de la partícula exploradora;

M —— masa del punto-masa generador del campo gravitacional;

$\hat{\varrho}$ —— cuadrivector de posición de un acontecimiento en la línea de universo de la partícula exploradora;

$\hat{\varrho}$ —— $\hat{\varrho}(t, x, y, z) = \hat{\varrho}(x^1, x^2, x^3, x^4)$;

$d\hat{\varrho}$ —— $d\hat{\varrho}(dt, dx, dy, dz) = d\hat{\varrho}(dx^1, dx^2, dx^3, dx^4)$;

$\delta\hat{\varrho}$ —— $\delta\hat{\varrho}(\delta t, \delta x, \delta y, \delta z) = \delta\hat{\varrho}(\delta x^1, \delta x^2, \delta x^3, \delta x^4)$

$\delta\hat{\varrho}$ es la variación libre.

$\hat{v}$ —— $\hat{v}\left(\frac{dt}{ds}, \frac{dx}{ds}, \frac{dy}{ds}, \frac{dz}{ds}\right) = \hat{v}\left(\frac{dx^1}{ds}, \frac{dx^2}{ds}, \frac{dx^3}{ds}, \frac{dx^4}{ds}\right)$

cuadrivector velocidad de la partícula exploradora.

$\hat{R}$ —— $\hat{R}(T, X, Y, Z) = \hat{R}(X^1, X^2, X^3, X^4)$

cuadrivector de posición de un acontecimiento en la línea de universo del punto masa generador del campo; esta posición es la posición retardada con respecto al ~~la posici~~ acontecimiento (t, x, y, z) de la línea de universo de la partícula exploradora.

~~$\hat{V}\left(\frac{dT}{dS}\right.$~~

S —— Longitud de arco de Minkowski en la línea de universo del punto masa generador del campo. Esa longitud de arco se cuenta a partir de un origen de los arcos.

dS —— $dS^2 = dT^2 - dX^2 - dY^2 - dZ^2$;

$dS^2 = \Delta_{ij}\, dX^i dX^j$.

$dS^2 = (dX^1)^2 - (dX^2)^2 - (dX^3)^2 - (dX^4)^2$.

$\hat{V}$ —— Cuadrivector velocidad retardada del punto masa generador del campo.

$\hat{V}\left(\frac{dT}{dS}, \frac{dX}{dS}, \frac{dY}{dS}, \frac{dZ}{dS}\right)$.

$\hat{V}$ —— La velocidad $\hat{V}$ es la que tiene el punto masa g. d. c. en el acontecimiento $(T, X, Y, Z.)$.

~~$\hat{A}$~~

$\hat{r}$ —— $\hat{\xi} - \hat{R}$, cuadrivector que liga ~~la~~ el acontecimiento ~~posición~~ retardado del punto masa generador del campo con el acontecimiento de la línea ~~la partícula~~ de universo de la partícula exploradora.

$$\hat{r} = \hat{\xi} - \hat{R}.$$

$$\cancel{\hat{r}(x - X}$$

$$\hat{r}(t-T, x-X, y-Y, z-Z).$$

$\hat{A}$ —— cuadrivector aceleración retardada del punto masa generador del campo.

$$\hat{A} = \frac{d\hat{V}}{dS}.$$

$\hat{A}$ —— cuadrivector aceleración del punto masa generador del campo en el acontecimiento (T, X, Y, Z).

Se introduce la integral I:

$$I = \int_a^b e^{\frac{2M}{(\hat{r}\cdot\hat{V})}} \left[2\hat{V} - \frac{\hat{v}}{(\hat{V}\cdot\hat{v})} \right] \cdot d\hat{s} \, .$$

La condición $\delta J = 0$ es equivalente a la condición $\delta I = 0$.

Cálculo de la variación de I.

Para calcular la variación de I hay que calcular las siguientes variaciones:

1) $\delta(ds)$;

2) δS;

3) $\delta \hat{r}$;

4) $\delta \hat{v}$;

5) $\delta \hat{V}$;

6) $\delta(\hat{r}\cdot\hat{V})$;

7) $\delta \frac{1}{(\hat{r}\cdot\hat{V})}$;

8) $\delta(\hat{V}\cdot\hat{v})$;

~~9) $\delta\left[2\hat{V} - \frac{\hat{v}}{(\hat{V}\cdot\hat{v})}\right]\cdot d\hat{s}$;~~

1) $\delta(ds)$

$$ds = \sqrt{d\hat{\xi}\cdot d\hat{\xi}} = \sqrt{d\hat{\xi}^{2}}$$

$$\cancel{\delta(ds) = \frac{d(\delta\hat{\xi}^{2})}{\sqrt{d\hat{\xi}^{2}}} = \frac{d}{ds}(\delta\hat{\xi})}$$

$$\delta(ds) = \frac{d\hat{\xi}\cdot d(\delta\hat{\xi})}{\sqrt{d\hat{\xi}^{2}}} = \frac{d\hat{\xi}\cdot d(\delta\hat{\xi})}{ds}$$

$$\boxed{\delta(ds) = \hat{v}\cdot d(\delta\hat{\xi})}$$ ✓ Comprobada.

2) δS

Todas las magnitudes retardadas son funciones de S.

El cuadrivector $\hat{r} = \hat{\xi} - \hat{R}$ es de longitud nula. (Liga a un acontecimiento de la línea de universo de la partícula exploradora con el acontecimiento retardado correspondiente de la línea de universo del punto masa generador del campo).

$$\hat{r}\cdot\hat{r} = 0$$

$$\cancel{(\hat{\xi} - \hat{R})^{2} = 0}$$

$$\hat{r}\cdot\delta\hat{r}=0$$

$$\hat{r}\cdot\left(\delta\hat{\rho}-\frac{d\hat{R}}{dS}\delta S\right)=0$$

$$\hat{r}\cdot\delta\hat{\rho}-\hat{r}\cdot\hat{V}\delta S=0$$

$$\boxed{\delta S=\frac{\hat{r}\cdot\delta\hat{\rho}}{\hat{r}\cdot\hat{V}}}$$ ✓ comprobada

3) $\delta\hat{r}$

$$\hat{r}=\hat{\rho}-\hat{R}$$

$$\delta\hat{r}=\delta\hat{\rho}-\hat{V}\frac{\hat{r}\cdot\delta\hat{\rho}}{\hat{r}\cdot\hat{V}}$$

$$\boxed{\delta\hat{r}=\delta\hat{\rho}-\frac{\hat{r}\cdot\delta\hat{\rho}}{\hat{r}\cdot\hat{V}}\hat{V}}$$ ✓ comprobada

4) $\delta\hat{v}$

$$\hat{v}=\frac{d\hat{\rho}}{ds}$$

$$\delta\hat{v}=\frac{d}{ds}(\delta\hat{\rho})-\frac{\hat{v}\cdot d(\delta\hat{\rho})}{ds^2}d\hat{\rho}$$

$$\delta \hat{v} = \frac{d}{ds}(\delta\hat{\varrho}) - \left\{\hat{v}\cdot\frac{d}{ds}(\delta\hat{\varrho})\right\}\hat{v}$$

$$\boxed{\delta \hat{v} = \frac{d}{ds}(\delta\hat{\varrho}) - \left\{\hat{v}\cdot\frac{d}{ds}(\delta\hat{\varrho})\right\}\hat{v}}$$ Comprobada.

5) $\delta\hat{V}$

$$\delta\hat{V} = \hat{A}\,\delta S = \hat{A}\,\frac{\hat{r}\cdot\delta\hat{\varrho}}{\hat{r}\cdot\hat{V}}$$

$$\boxed{\delta\hat{V} = \frac{\hat{r}\cdot\delta\hat{\varrho}}{\hat{r}\cdot\hat{V}}\hat{A}}$$ Comprobada...

6) $\delta(\hat{r}\cdot\hat{V})$

$$\delta(\hat{r}\cdot\hat{V}) = \hat{r}\cdot\delta\hat{V} + \hat{V}\cdot\delta\hat{r}$$

$$\delta(\hat{r}\cdot\hat{V}) = (\hat{r}\cdot\hat{A})\frac{\hat{r}\cdot\delta\hat{\varrho}}{\hat{r}\cdot\hat{V}} + \hat{V}\cdot\delta\hat{\varrho} - \frac{\hat{r}\cdot\delta\hat{\varrho}}{\hat{r}\cdot\hat{V}}$$

$$\boxed{\delta(\hat{r}\cdot\hat{V}) = \left\{\hat{r}\cdot\hat{A} - 1\right\}\frac{\hat{r}\cdot\delta\hat{\varrho}}{\hat{r}\cdot\hat{V}} + \hat{V}\cdot\delta\hat{\varrho}}$$ Comprob.

7) $\delta \dfrac{1}{\hat{r}\cdot\hat{V}}$

$$\delta \frac{1}{\hat{r}\cdot\hat{V}} = \frac{-1}{(\hat{r}\cdot\hat{V})^2}\left[\left\{\hat{r}\cdot\hat{A}-1\right\}\frac{\hat{r}\cdot\delta\hat{\varrho}}{\hat{r}\cdot\hat{V}} + \hat{V}\cdot\delta\hat{\varrho}\right]$$

$$\boxed{\delta \frac{1}{\hat{r}\cdot\hat{V}} = -\left\{\hat{r}\cdot\hat{A}-1\right\}\frac{\hat{r}\cdot\delta\hat{\varrho}}{(\hat{r}\cdot\hat{V})^3} - \frac{\hat{V}\cdot\delta\hat{\varrho}}{(\hat{r}\cdot\hat{V})^2}}$$

Comprobado

8) $\delta(\hat{V}\cdot\hat{\nu})$

$$\boxed{\delta(\hat{V}\cdot\hat{\nu}) = \frac{\hat{r}\cdot\delta\hat{\varrho}}{\hat{r}\cdot\hat{V}}\,\hat{A}\cdot\hat{\nu} + \hat{V}\cdot\frac{d}{ds}(\delta\hat{\varrho}) - \left\{\hat{\nu}\cdot\frac{d}{ds}(\delta\hat{\varrho})\right\}(\hat{\nu}\cdot\hat{V})}$$

Comprobada

9) $\delta(\cancel{\hat{V}\cdot d\hat{\varrho}})$

$$\delta\, e^{\frac{2M}{\hat{r}\cdot\hat{V}}}\left[2\hat{V} - \frac{\hat{v}}{\hat{V}\cdot\hat{v}}\right]\cdot d$$

$$\Bigg[\Bigg\{2\,\frac{\hat{r}\cdot\delta\hat{\rho}}{\hat{r}\cdot\hat{V}}\,\hat{A} + \frac{1}{(\hat{V}\cdot\hat{v})^2}\Big\langle \frac{\hat{r}\cdot\delta\hat{\rho}}{\hat{r}\cdot\hat{V}}\,\hat{A}\cdot\hat{v} + \hat{V}\cdot\frac{d}{ds}(\delta\hat{\rho}) \quad - \Big\{\hat{v}\cdot\frac{d}{ds}(\delta\hat{\rho})$$

$$\left[2\hat{V} - \frac{\hat{v}}{\hat{V}\cdot\hat{v}}\right]\left\{-2M\left\{\hat{r}\cdot\hat{A}-1\right\}\frac{\hat{r}\cdot\delta\hat{\rho}}{(\hat{r}\cdot\hat{V})^3} - 2M\,\frac{\hat{V}\cdot\delta\hat{\rho}}{(\hat{r}\cdot\hat{V})^2}\right\}\Bigg]$$

Iguales y d

mismo sign

$$e^{\frac{2M}{\hat{r}\cdot\hat{V}}}\Bigg[+ \left[2\hat{V} - \frac{\hat{v}}{\hat{V}\cdot\hat{v}}\right]\cdot d(\delta\hat{\rho})\Bigg]$$

$$\delta\, e^{\frac{2M}{\hat{r}\cdot\hat{V}}}\left[2\hat{V} - \frac{\hat{v}}{\hat{V}\cdot\hat{v}}\right]\cdot d\hat{\xi}$$

Comprobado

$$\cdot\hat{v} + \hat{V}\cdot\frac{d}{ds}(\delta\hat{\xi}) - \left\{\hat{v}\cdot\frac{d}{ds}(\delta\hat{\xi})\right\}(\hat{v}\cdot\hat{V})\Big\rangle\hat{v} - \frac{\frac{d}{ds}(\delta\hat{\xi})}{\hat{V}\cdot\hat{v}} + \frac{\left\{\hat{v}\cdot\frac{d}{ds}(\delta\hat{\xi})\right\}\hat{v}}{\hat{V}\cdot\hat{v}}\Big\}\cdot d\hat{\xi}$$

$$\hat{A} - 1\Big\}\frac{\hat{r}\cdot\delta\hat{\xi}}{(\hat{r}\cdot\hat{V})^3} - 2M\,\frac{\hat{V}\cdot\delta\hat{\xi}}{(\hat{r}\cdot\hat{V})^2}\Big\}$$

¡estos dos términos se anulan!

iguales y del mismo signo

$$\delta e^{\frac{2M}{\hat{r}\cdot\hat{V}}}\left[2\hat{V} - \frac{\hat{v}}{\hat{V}\cdot\hat{v}}\right]\cdot d\hat{\rho}$$

menos

$$e^{\frac{2M}{\hat{r}\cdot\hat{V}}}\Bigg[\Bigg\langle 2\frac{\hat{A}\cdot d\hat{\rho}}{\hat{r}\cdot\hat{V}}\hat{r} + \frac{(\hat{A}\cdot\hat{v})(\hat{v}\cdot d\hat{\rho})}{(\hat{r}\cdot\hat{V})(\hat{V}\cdot\hat{v})^2}\hat{r} - \{2M\}\Big(2\hat{V}\cdot d\hat{\rho} - \frac{\hat{v}\cdot}{\hat{V}\cdot}$$

$$+ \cancel{\left[2\hat{V} + \frac{1}{(\hat{V}\cdot\hat{v})^2}\hat{V}\cdot\frac{d}{ds}(\delta\hat{\rho})\right](\hat{v}\cdot d\hat{\rho})}$$

$$+ \cancel{\left[2\hat{V}\right.} * \qquad -$$

$$+ \left[2\hat{V} - \frac{2\hat{v}}{\hat{V}\cdot\hat{v}}\right]\cdot d(\delta\hat{\rho}) + \frac{1}{(\hat{V}\cdot\hat{v})^2}\left[\hat{V}\cdot\frac{d}{ds}(\delta\hat{\rho})\right]\Big(\hat{v}$$

$$\Bigg]$$

$$\left[\hat{V}\cdot\frac{d}{ds}(\delta\hat{\rho})\right](\hat{v}\cdot d$$

$$\hat{V} - \frac{\hat{v}}{\hat{V}\cdot\hat{v}}\Big] \cdot d\hat{\varrho}$$

menos

$$\{2M\}\Big\{2\hat{V}\cdot d\hat{\varrho} - \frac{\hat{v}\cdot d\hat{\varrho}}{\hat{V}\cdot\hat{v}}\Big]\Big[\{\hat{r}\cdot\hat{A}-1\}\frac{\hat{r}}{(\hat{r}\cdot\hat{V})^3} + \frac{\hat{V}}{(\hat{r}\cdot\hat{V})^2}\Big]\Big\} \cdot \delta\hat{\varrho}$$

$$(\hat{v}\cdot d\hat{\varrho}) \qquad \cancel{\frac{2\hat{v}}{\hat{V}\cdot\hat{v}}}$$

$$\frac{1}{(\hat{V}\cdot\hat{v})^2}\Big[\hat{V}\cdot\frac{d}{ds}(\delta\hat{\varrho})\Big]\Big[\hat{v}\cdot d\hat{\varrho}\Big]$$

$$\Big[\hat{V}\cdot\frac{d}{ds}(\delta\hat{\varrho})\Big](\hat{v}\cdot d\hat{\varrho}) = \Big[\hat{V}\cdot d(\delta\hat{\varrho})\Big]\ \hat{v}\cdot\Big(\frac{d\hat{\varrho}}{ds}\Big) = \hat{V}\cdot d(\delta\hat{\varrho})$$

$$\delta\, e^{\frac{2M}{\hat{r}\cdot\hat{V}}}\left[2\hat{V} - \frac{\hat{v}}{\hat{V}\cdot\hat{v}}\right]\cdot$$

Comprobado

Comprobado

$$e^{\frac{2M}{\hat{r}\cdot\hat{V}}}\left[\left\langle 2\,\frac{\hat{A}\cdot d\hat{\varrho}}{\hat{r}\cdot\hat{V}}\,\hat{r} + \frac{(\hat{A}\cdot\hat{v})(\hat{v}\cdot d\hat{\varrho})}{(\hat{r}\cdot\hat{V})(\hat{V}\cdot\hat{v})^2}\,\hat{r} - 2M\left\{2\hat{V}\cdot d\hat{\varrho} - \frac{\hat{v}\cdot d\hat{\varrho}}{\hat{V}\cdot\hat{v}}\right.\right.\right.$$

Comprobado Comprobado Comprobado

$$+\left[2\hat{V} - \frac{2\hat{v}}{\hat{V}\cdot\hat{v}} + \frac{\hat{V}}{(\hat{V}\cdot\hat{v})^2}\right]\cdot d(\delta\hat{\varrho})$$

$$-$$

$$\left\{-4M(\hat{V}\cdot\hat{v}) + \frac{2M}{(\hat{V}\cdot\hat{v})}\right\}\left\{\frac{\hat{r}\cdot\hat{A}}{(\hat{r}\cdot\hat{V})^3}\,\hat{r} - \frac{1}{(\hat{r}\cdot\hat{V})^3}\,\hat{r} + \frac{1}{(\hat{r}\cdot\hat{V})}\right.$$

$$-4M\,\frac{(\hat{V}\cdot\hat{v})(\hat{r}\cdot\hat{A})}{(\hat{r}\cdot\hat{V})^3}\,\hat{r} + \frac{4M(\hat{V}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})^3}\,\hat{r} + \frac{2M(\hat{r}\cdot\hat{A})}{(\hat{r}\cdot\hat{V})^3(\hat{V}\cdot\hat{v})}\,\hat{r} -$$

$$- \frac{\hat{v}}{\hat{V}\cdot\hat{v}}\Big]\cdot d\hat{\varrho}$$

Comprobado!

$$\cdot d\hat{\varrho} - \frac{\hat{v}\cdot d\hat{\varrho}}{\hat{V}\cdot\hat{v}}\Big\}\Big[\{\hat{r}\cdot\hat{A}-1\}\frac{\hat{r}}{(\hat{r}\cdot\hat{V})^3} + \frac{\hat{V}}{(\hat{r}\cdot\hat{V})^2}\Big]\Big\rangle\cdot\delta\hat{\varrho}$$

$$-$$

$$\hat{r} + \frac{1}{(\hat{r}\cdot\hat{V})^2}\hat{V}\Big\}$$

$$\frac{\cdot\hat{A})}{)^3(\hat{V}\cdot\hat{v})}\hat{r} - \frac{2M}{(\hat{r}\cdot\hat{V})^3(\hat{V}\cdot\hat{v})}\hat{r} - \frac{4M(\hat{V}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})^2}\hat{V} + \frac{2M}{(\hat{r}\cdot\hat{V})^2(\hat{V}\cdot\hat{v})}\hat{V}$$

Ecuación de las extremales:

$$\frac{d}{ds}\left[e^{\frac{2M}{\hat{r}\cdot\hat{V}}}\left[2\hat{V}-\frac{2\hat{v}}{\hat{V}\cdot\hat{v}}+\frac{\hat{V}}{(\hat{V}\cdot\hat{v})^2}\right]\right]=\left(2\frac{\hat{A}\cdot\hat{v}}{\hat{r}\cdot\hat{V}}\hat{r}+\frac{\hat{A}\cdot\hat{v}}{\hat{r}\cdot\hat{V}}\ \frac{}{(\hat{V}\cdot\hat{v}}\right.$$

$$\boxed{\frac{dS}{ds}}$$

$$\hat{r}^2=0$$

$$\hat{r}\cdot\hat{r}=0\ ;\quad \hat{r}=\hat{\rho}-\hat{R}$$

$$\hat{r}\cdot\left(\hat{v}-\hat{V}\frac{dS}{ds}\right)=0$$

$$\hat{r}\cdot\hat{v}-\hat{r}\cdot\hat{V}\frac{dS}{ds}=0$$

$$\boxed{\frac{dS}{ds}=\frac{\hat{r}\cdot\hat{v}}{\hat{r}\cdot\hat{V}}}$$

$$\boxed{\frac{d}{ds}\frac{2M}{\hat{r}\cdot\hat{V}}}$$

$$\frac{d}{ds}\frac{2M}{\hat{r}\cdot\hat{V}}=\frac{-2M\left((\hat{r}\cdot\hat{A})\frac{\hat{r}\cdot\hat{v}}{\hat{r}\cdot\hat{V}}+\hat{V}\cdot\left(\hat{v}-\hat{V}\frac{\hat{r}\cdot\hat{v}}{\hat{r}\cdot\hat{V}}\right)\right)}{(\hat{r}\cdot\hat{V})^2}$$

$$\frac{\hat{r}\cdot\hat{V}}{\hat{V}} \frac{\quad}{(\hat{V}\cdot\hat{v})^2}\hat{r} - 2M\left\{2\hat{V}\cdot\hat{v} - \frac{1}{\hat{V}\cdot\hat{v}}\right\}\left[\left\{\hat{r}\cdot\hat{A} - 1\right\}\frac{\hat{r}}{(\hat{r}\cdot\hat{V})^3} + \frac{\hat{V}}{(\hat{r}\cdot\hat{V})^2}\right]\Big\rangle e^{\frac{2M}{\hat{r}\cdot\hat{V}}}$$

$$\boxed{\frac{d}{ds}\frac{2M}{\hat{r}\cdot\hat{V}} = -2M\frac{(\hat{r}\cdot\hat{A})(\hat{r}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})^3} - \frac{2M(\hat{V}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})^2} + \frac{2M(\hat{r}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})^3}}$$

$$\boxed{\frac{d}{ds}\hat{V} =}$$

$$\boxed{\frac{d}{ds}\hat{V} = \hat{A}\frac{\hat{r}\cdot\hat{v}}{\hat{r}\cdot\hat{V}}}$$

$$\boxed{\frac{d}{ds}(\hat{V}\cdot\hat{v})}$$

$$\boxed{\frac{d}{ds}(\hat{V}\cdot\hat{v}) = \hat{V}\cdot\hat{a} + (\hat{v}\cdot\hat{A})\frac{\hat{r}\cdot\hat{v}}{\hat{r}\cdot\hat{V}}}$$

$$\boxed{\frac{d}{ds}\frac{\hat{v}}{(\hat{V}\cdot\hat{v})}}$$

$$\frac{d}{ds}\frac{\hat{v}}{(\hat{V}\cdot\hat{v})} = \frac{\hat{a}}{\hat{V}\cdot\hat{v}} - \frac{\hat{V}\cdot\hat{a}}{(\hat{V}\cdot\hat{v})^2}\hat{v} - \frac{(\hat{v}\cdot\hat{A})(\hat{r}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})(\hat{V}\cdot\hat{v})^2}\hat{v}$$

$$\boxed{\frac{d}{ds}\frac{\hat{v}}{(\hat{V}\cdot\hat{v})} = \frac{\hat{a}}{\hat{V}\cdot\hat{v}} - \frac{\hat{V}\cdot\hat{a}}{(\hat{V}\cdot\hat{v})^2}\hat{v} - \frac{(\hat{v}\cdot\hat{A})(\hat{r}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})(\hat{V}\cdot\hat{v})^2}\hat{v}}$$

$$\cancel{\frac{d}{ds}\frac{\hat{V}}{(\hat{V}\cdot\hat{v})^2} = \hat{r}\cdot\hat{V}}$$

$$\boxed{\frac{d}{ds}\frac{\hat{V}}{(\hat{V}\cdot\hat{v})^2}}$$

$$\boxed{\frac{d}{ds}\frac{\hat{V}}{(\hat{V}\cdot\hat{v})^2} = \left[\frac{(\hat{r}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})(\hat{V}\cdot\hat{v})^2}\hat{A} - \frac{2(\hat{V}\cdot\hat{a})}{(\hat{V}\cdot\hat{v})^3}\hat{V} - \frac{2(\hat{v}\cdot\hat{A})(\hat{r}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})(\hat{V}\cdot\hat{v})^3}\hat{V}\right]}$$

Coeficiente de $\hat{r}$

$$-2\frac{\hat{A}\cdot\hat{v}}{\hat{r}\cdot\hat{V}} \mp \frac{\hat{A}\cdot\hat{v}}{(\hat{r}\cdot\hat{V})(\hat{V}\cdot\hat{v})^2} +$$

$$+2M\left\{2(\hat{V}\cdot\hat{v}) - \frac{1}{\hat{V}\cdot\hat{v}}\right\}\left\{\frac{\hat{r}\cdot\hat{A}-1}{(\hat{r}\cdot\hat{V})^3}\right\}.$$

Coeficiente de $\hat{a}$.

$$-\frac{2}{(\hat{V}\cdot\hat{v})}$$

Coeficiente de $\hat{v}$

$$+\frac{4M(\hat{r}\cdot\hat{A})(\hat{r}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})^3(\hat{V}\cdot\hat{v})} + \frac{4M\cancel{(\hat{V}\cdot\hat{v})}}{(\hat{r}\cdot\hat{V})^2\cancel{(\hat{V}\cdot\hat{v})}} - \frac{4M(\hat{r}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})^3(\hat{V}\cdot\hat{v})}$$

$$+2\frac{\hat{V}\cdot\hat{a}}{(\hat{V}\cdot\hat{v})^2} + 2\frac{(\hat{v}\cdot\hat{A})(\hat{r}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})(\hat{V}\cdot\hat{v})^2}$$

$$\boxed{\begin{aligned}&\frac{4M(\hat{r}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})^3(\hat{V}\cdot\hat{v})}\left[\hat{r}\cdot\hat{A}-1\right] + \frac{4M}{(\hat{r}\cdot\hat{V})^2}\\ &+2\frac{\hat{V}\cdot\hat{a}}{(\hat{V}\cdot\hat{v})^2} + 2\frac{(\hat{v}\cdot\hat{A})(\hat{r}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})(\hat{V}\cdot\hat{v})^2}\end{aligned}}$$

Coeficiente de $\hat{V}$.

$$-4M\frac{(\hat r\cdot\hat A)(\hat r\cdot\hat v)}{(\hat r\cdot\hat V)^3} - \frac{4M(\hat V\cdot\hat v)}{(\hat r\cdot\hat V)^2} + \frac{4M(\hat r\cdot\hat v)}{(\hat r\cdot\hat V)^3}$$

$$-\frac{2M(\hat r\cdot\hat A)(\hat r\cdot\hat v)}{(\hat r\cdot\hat V)^3(\hat V\cdot\hat v)^2} - \frac{2M\cancel{(\hat V\cdot\hat v)}}{(\hat r\cdot\hat V)^2(\hat V\cdot\hat v)^{\cancel{2}}} + \frac{2M(\hat r\cdot\hat v)}{(\hat r\cdot\hat V)^3(\hat V\cdot\hat v)^2}$$

$$-\frac{2(\hat V\cdot\hat a)}{(\hat V\cdot\hat v)^3} - \frac{2(\hat v\cdot\hat A)(\hat r\cdot\hat v)}{(\hat r\cdot\hat V)(\hat V\cdot\hat v)^3}$$

$$+\frac{2M}{(\hat r\cdot\hat V)^2}\left\{2\hat V\cdot\hat v - \cancel{\frac{1}{\hat V\cdot\hat v}}\right\}$$

$$-\frac{4M(\hat r\cdot\hat A)(\hat r\cdot\hat v)}{(\hat r\cdot\hat V)^3} + \frac{4M(\hat r\cdot\hat v)}{(\hat r\cdot\hat V)^3}$$

$$-\frac{2M(\hat r\cdot\hat A)(\hat r\cdot\hat v)}{(\hat r\cdot\hat V)^3(\hat V\cdot\hat v)^2} - \frac{4M}{(\hat r\cdot\hat V)^2(\hat V\cdot\hat v)}$$

$$+\frac{2M(\hat r\cdot\hat v)}{(\hat r\cdot\hat V)^3(\hat V\cdot\hat v)^2} - \frac{2(\hat V\cdot\hat a)}{(\hat V\cdot\hat v)^3}$$

$$+\frac{2(\hat v\cdot\hat A)(\hat r\cdot\hat v)}{(\hat r\cdot\hat V)(\hat V\cdot\hat v)^3}$$

Coeficiente de $\hat{A}$

$$\boxed{2\,\frac{\hat{r}\cdot\hat{v}}{\hat{r}\cdot\hat{V}} + \frac{\hat{r}\cdot\hat{v}}{(\hat{r}\cdot\hat{V})(\hat{V}\cdot\hat{v})^2}}$$

Relación entre los 5 cuadrivectores $\hat{r}, \hat{v}, \hat{a}; \hat{V}, \hat{A}$.

$$\left[-2\frac{\hat{A}\cdot\hat{v}}{\hat{r}\cdot\hat{V}} - \frac{\hat{A}\cdot\hat{v}}{(\hat{r}\cdot\hat{V})(\hat{V}\cdot\hat{v})^2} + 2M\left\{2(\hat{V}\cdot\hat{v}) - \frac{1}{\hat{V}\cdot\hat{v}}\right\}\frac{\hat{r}\cdot\hat{A}-1}{(\hat{r}\cdot\hat{V})^3}\right]\hat{r}$$

$$+\left[\frac{4M(\hat{r}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})^3(\hat{V}\cdot\hat{v})}\left[\hat{r}\cdot\hat{A}-1\right] + \frac{4M}{(\hat{r}\cdot\hat{V})^2} + 2\frac{\hat{V}\cdot\hat{a}}{(\hat{V}\cdot\hat{v})^2} + 2\frac{(\hat{v}\cdot\hat{A})(\hat{r}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})(\hat{V}\cdot\hat{v})^2}\right]\hat{v}$$

$$-\frac{2}{(\hat{V}\cdot\hat{v})}\hat{a}$$

$$+\left[-\frac{4M(\hat{r}\cdot\hat{A})(\hat{r}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})^3} + \frac{4M(\hat{r}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})^3} - \frac{2M(\hat{r}\cdot\hat{A})(\hat{r}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})^3(\hat{V}\cdot\hat{v})^2} + \right.$$

$$\left. - \frac{4M}{(\hat{r}\cdot\hat{V})^2(\hat{V}\cdot\hat{v})} + \frac{2M(\hat{r}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})^3(\hat{V}\cdot\hat{v})^2} - \frac{2(\hat{V}\cdot\hat{a})}{(\hat{V}\cdot\hat{v})^3}\right]\hat{V}$$

$$+\left[-\frac{4M(\hat{r}\cdot\hat{A})(\hat{r}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})^3}+\frac{4M(\hat{r}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})^3}-\frac{2M(\hat{r}\cdot\hat{A})(\hat{r}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})^3(\hat{V}\cdot\hat{v})^2}+\right.$$

$$\left.-\frac{4M}{(\hat{r}\cdot\hat{V})^2(\hat{V}\cdot\hat{v})}+\frac{2M(\hat{r}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})^3(\hat{V}\cdot\hat{v})^2}-\frac{2(\hat{V}\cdot\hat{a})}{(\hat{V}\cdot\hat{v})^3}+\frac{2(\hat{v}\cdot\hat{A})(\hat{r}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})(\hat{V}\cdot\hat{v})^3}\right]\hat{V}$$

$$+\left[2\,\frac{\hat{r}\cdot\hat{v}}{\hat{r}\cdot\hat{V}}+\frac{\hat{r}\cdot\hat{v}}{(\hat{r}\cdot\hat{V})(\hat{V}\cdot\hat{v})^2}\right]\hat{A}$$

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$$2\frac{(\hat{V}\cdot\hat{a})}{(\hat{V}\cdot\hat{v})^2} = \Bigg[-\frac{2M(\hat{r}\cdot\hat{A})}{(\hat{r}\cdot\hat{V})^2(\hat{V}\cdot\hat{v})^2} + \frac{4M(\hat{r}\cdot\hat{A})}{(\hat{r}\cdot\hat{V})^2} - \frac{2M(\hat{r}\cdot\hat{v})(\hat{r}\cdot\hat{A})}{(\hat{r}\cdot\hat{V})^3(\hat{V}\cdot\hat{v})}$$

$$-\frac{8M(\hat{v}\cdot\hat{A})}{(\hat{r}\cdot\hat{V})(\hat{V}\cdot\hat{v})} + \frac{4M(\hat{r}\cdot\hat{v})(\hat{v}\cdot\hat{A})}{(\hat{r}\cdot\hat{V})^2(\hat{V}\cdot\hat{v})^2} + \frac{2M(\hat{r}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})^3(\hat{V}\cdot\hat{v})}$$

$$+\frac{2M}{(\hat{r}\cdot\hat{V})^2} \Bigg]$$

$$\cancel{\hat{r}^2 = 0} \qquad \cancel{\frac{d}{ds}(\hat{r}^2) = 0}$$

$$\cancel{\hat{r}\cdot\hat{V}} \qquad \cancel{\hat{r}\cdot\hat{V} = 0}$$

$$\hat{r} = 0$$

$$(\hat{\rho} - \hat{R})\cdot\hat{r} = 0$$

$$\left(\hat{v} - \hat{V}\frac{dS}{ds}\right)\cdot\hat{r} = 0$$

$$\hat{r}\cdot\hat{v} - \hat{r}\cdot\hat{V}\frac{dS}{ds} = 0$$

$$\boxed{\frac{dS}{ds} = \frac{r\cdot\hat{v}}{\hat{r}\cdot\hat{V}}}$$

$$(\hat{V}\cdot\hat{a}) = \Bigg[\frac{M}{(\hat{r}\cdot\hat{V})^2} - \frac{M(\hat{r}\cdot\hat{A})}{(\hat{r}\cdot\hat{V})^2} - \frac{2M(\hat{v}\cdot\hat{V})^2}{(\hat{r}\cdot\hat{V})^2} - \frac{4M(\hat{v}\cdot\hat{V})(\hat{v}\cdot\hat{A})}{(\hat{r}\cdot\hat{V})} + \frac{2M(\hat{r}\cdot\hat{A})(\hat{v}\cdot\hat{V})^2}{(\hat{r}\cdot\hat{V})^3}$$

$$- \frac{M(\hat{v}\cdot\hat{V})(\hat{r}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})^3} + \frac{M(\hat{v}\cdot\hat{V})(\hat{r}\cdot\hat{v})(\hat{r}\cdot\hat{A})}{(\hat{r}\cdot\hat{V})^3} + \frac{M(\hat{v}\cdot\hat{V})^2}{(\hat{r}\cdot\hat{V})^2}$$

$$- \frac{M}{(\hat{r}\cdot\hat{V})^2} + \frac{2M(\hat{r}\cdot\hat{v})(\hat{v}\cdot\hat{V})}{(\hat{r}\cdot\hat{V})^3} + \frac{2M(\hat{r}\cdot\hat{v})(\hat{v}\cdot\hat{A})}{(\hat{r}\cdot\hat{V})^2} - \frac{2M(\hat{r}\cdot\hat{v})(\hat{r}\cdot\hat{A})(\hat{v}\cdot\hat{V})}{(\hat{r}\cdot\hat{V})^3}$$

$$-$$

Nuevo Plan.

Ensayar la integral de acción

$$J = \int_a^b e^{\frac{2}{(\hat{r}\cdot\hat{V})}} \left[2\hat{V} - \frac{\hat{V}}{(\hat{V}\cdot\hat{V})} \right] \cdot \left(d\hat{s} - d\hat{R} \right)$$

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Falta calcular $\delta d\hat{R}$

$$d\hat{R} = \hat{V}\, dS$$

$$\delta d\hat{R} = \hat{V}\, \delta(dS) + (\delta\hat{V})\, dS$$

$$\delta(d\hat{R}) = \hat{V}\, \delta(dS) + \frac{\hat{r}\cdot\delta\hat{s}}{\hat{r}\cdot\hat{V}} \hat{A}\, dS$$

$$\delta(d\hat{R}) = \hat{V}\, \delta(dS) + \frac{\hat{r}\cdot\delta\hat{s}}{(\hat{r}\cdot\hat{V})} d\hat{V}$$

$$\boxed{\frac{dS}{ds} = ?}$$

$$\hat{r}\cdot\hat{r} = 0$$

$$\hat{r}\cdot d\hat{r} = 0$$

Como $\hat{r} = \hat{\rho} - \hat{R}$

$$d\hat{r} = d\hat{\rho} - d\hat{R}$$

$$\therefore\ \hat{r}\cdot(d\hat{\rho} - d\hat{R}) = 0$$

$$\hat{r}\cdot(\hat{v}\,ds - \hat{V}\,dS) = 0$$

$$\left.\begin{aligned}&\hat{r}\cdot\left[\hat{v}\,\delta(ds) + (\delta\hat{v})ds - \hat{V}\,\delta(dS) - dS\,\delta\hat{V}\right]\\ &+\delta\hat{r}\cdot\left[\hat{v}\,ds - \hat{V}\,dS\right]\end{aligned}\right\} = 0$$

$$\left.\begin{aligned}&(\hat{r}\cdot\hat{v})(\hat{v}\cdot d\,\delta\hat{\rho}) + \cancel{(\hat{r}\cdot\delta\hat{v})ds} + \hat{r}\cdot d(\delta\hat{\rho}) - (\hat{r}\cdot\hat{v})\{\hat{v}\cdot d\delta\hat{\rho}\}\\ &-(\hat{r}\cdot\hat{V})\,\delta\, dS - (\hat{r}\cdot\hat{A})\frac{\hat{r}\cdot\delta\hat{\rho}}{\hat{r}\cdot\hat{V}}\,\frac{(\hat{r}\cdot\hat{v})}{\hat{r}\cdot\hat{V}}\,ds\\ &\cancel{+(\hat{v}\cdot\delta\hat{\rho})ds - \hat{V}\cdot\delta} + \left[\delta\hat{\rho} - \hat{V}\frac{\hat{r}\cdot\delta\hat{\rho}}{\hat{r}\cdot\hat{V}}\right](\hat{v}\,ds - \hat{V}\,dS)\end{aligned}\right\} = 0$$

$$\left.\begin{aligned}&\cancel{(\hat{r}\cdot\hat{v})(\hat{v}\cdot d\,\delta\hat{\rho})} + \hat{r}\cdot d(\delta\hat{\rho}) - \cancel{(\hat{r}\cdot\hat{v})(\hat{v}\cdot d\delta\hat{\rho})}\\ &-(\hat{r}\cdot\hat{V})\,\delta\, dS - (\hat{r}\cdot\hat{A})\frac{\hat{r}\cdot\delta\hat{\rho}}{(\hat{r}\cdot\hat{V})}\,\frac{(\hat{r}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})}\,ds\\ &+\hat{v}\cdot\delta\hat{\rho}\,ds - (\hat{V}\cdot\delta\hat{\rho})\frac{(\hat{r}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})}ds - (\hat{V}\cdot\hat{v})\frac{\hat{r}\cdot\delta\hat{\rho}}{(\hat{r}\cdot\hat{V})}ds + \frac{\hat{r}\cdot\delta\hat{\rho}}{(\hat{r}\cdot\hat{V})}\,\frac{(\hat{r}\cdot\hat{v})}{\hat{r}\cdot\hat{V}}ds\end{aligned}\right\} = 0$$

$$\underline{\underline{\delta dS}} = \left[\frac{\hat{r}\cdot d(\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})}\right] + \left[-\frac{(\hat{r}\cdot\hat{A})(\hat{r}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})^3}\hat{r} + \frac{\hat{v}}{(\hat{r}\cdot\hat{V})} - \frac{(\hat{r}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})^2}\hat{V} - \frac{(\hat{V}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})^2}\hat{r} + \frac{\hat{r}\cdot\hat{v}}{(\hat{r}\cdot\hat{V})^3}\hat{r} \right]\cdot\delta\hat{\rho}\, ds$$

$$\delta dS = \cancel{\frac{\hat{r}}{\hat{r}\cdot\hat{V}}\cdot d(\delta\hat{\rho}) + \left[+ \left\{\frac{\hat{r}\cdot\hat{v}}{(\hat{r}\cdot\hat{V})^3} - \frac{(\hat{r}\cdot\hat{v})(\hat{r}\cdot\hat{A})}{(\hat{r}\cdot\hat{V})^3}\right\}\hat{r} \right.}$$

$$\boxed{\delta dS = \frac{\hat{r}}{\hat{r}\cdot\hat{V}}\cdot d(\delta\hat{\rho}) + \left[\left\{ + \frac{\hat{r}\cdot\hat{v}}{(\hat{r}\cdot\hat{V})^3} - \frac{(\hat{r}\cdot\hat{v})(\hat{r}\cdot\hat{A})}{(\hat{r}\cdot\hat{V})^3} + \frac{(\hat{V}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})^2}\right\}\hat{r} - \frac{\hat{r}\cdot\hat{v}}{(\hat{r}\cdot\hat{V})^2}\hat{V} + \frac{1}{\hat{r}\cdot\hat{V}}\hat{v} \right]\cdot\delta\hat{\rho}\, ds}$$

Cálculo de:

$$\left[2\hat{V} - \frac{\hat{V}}{(\hat{V}\cdot\hat{V})}\right]\cdot d\hat{R} = dK.$$

$$dK = 2(\hat{V}\cdot\hat{V})dS - \frac{(\hat{V}\cdot\hat{V})}{(\hat{V}\cdot\hat{V})}dS$$

$$dK = [2-1]dS = dS.$$

$$\boxed{\left[2\hat{V} - \frac{\hat{V}}{(\hat{V}\cdot\hat{V})}\right]\cdot d\hat{R} = dS}$$

$$\boxed{J = \int_a^b e^{\frac{2}{(\hat{r}\cdot\hat{V})}}\left[\left\{2\hat{V} - \frac{\hat{V}}{\hat{V}\cdot\hat{V}}\right\}\cdot d\hat{\rho} - dS\right]}$$

$$\cancel{\hat{r} = \hat{\rho} - \hat{R}}$$

$$\cancel{\hat{r}\cdot\hat{r} = a}$$

$$\cancel{\hat{\rho}\cdot\hat{r} - \hat{R}\cdot\hat{r} = 0}$$

$$\hat{r}\cdot\hat{r} = 0$$

$$\hat{r}\cdot d\hat{r} = 0$$

$$\hat{r}\cdot[d\hat{\rho} - d\hat{R}] = 0$$

$$\hat{r}\cdot d\hat{\rho} - \hat{r}\cdot\hat{V}\,dS = 0$$

$$\boxed{\delta \frac{\hat{r}}{(\hat{r}\cdot\hat{V})} = \left[\frac{\delta\hat{\rho}}{(\hat{r}\cdot\hat{V})} - \frac{\hat{r}\cdot\delta\hat{\rho}}{(\hat{r}\cdot\hat{V})^2}\,\hat{V} - \left\{ \hat{r}\cdot\hat{A} - 1 \right\} \frac{\hat{r}\cdot\delta\hat{\rho}}{(\hat{r}\cdot\hat{V})^3}\,\hat{r} - \frac{\hat{V}\cdot\delta\hat{\rho}}{(\hat{r}\cdot\hat{V})^2}\,\hat{r} \right]}$$

$$\boxed{dS = \frac{\hat{r}}{(\hat{r}\cdot\hat{V})}\cdot d\hat{\varrho}}$$

$$\boxed{-J = \int_a^b e^{\frac{2}{(\hat{r}\cdot\hat{V})}}\left[2\hat{V} - \frac{\hat{v}}{(\hat{r}\cdot\hat{V})} - \frac{\hat{r}}{(\hat{r}\cdot\hat{V})}\right]\cdot d\hat{\varrho}}$$

· Ensayar la integral de acción:

$$-J = \int_a^b e^{\frac{2}{(\hat{r}\cdot\hat{V})}}\left[2\hat{V} - \frac{\hat{v}}{(\hat{r}\cdot\hat{V})} - \frac{\hat{r}}{(\hat{r}\cdot\hat{V})}\right]\cdot d\hat{\varrho}$$

$$\boxed{\delta\hat{r} = \delta\hat{\varrho} - \frac{\hat{r}\cdot\delta\hat{\varrho}}{\hat{r}\cdot\hat{V}}\hat{V}}$$ pág. 6.

$$\delta\frac{1}{(\hat{r}\cdot\hat{V})} = -\left\{\hat{r}\cdot\hat{A} - 1\right\}\frac{\hat{r}\cdot\delta\hat{\varrho}}{(\hat{r}\cdot\hat{V})^3} - \frac{\hat{V}\cdot\delta\hat{\varrho}}{(\hat{r}\cdot\hat{V})^2}$$ pág. 8.

$$\delta e^{\frac{2}{(\hat{r}\cdot\hat{V})}} = 2e^{\frac{2}{(\hat{r}\cdot\hat{V})}}\left[-\left\{\hat{r}\cdot\hat{A} - 1\right\}\frac{\hat{r}\cdot\delta\hat{\varrho}}{(\hat{r}\cdot\hat{V})^3} - \frac{\hat{V}\cdot\delta\hat{\varrho}}{(\hat{r}\cdot\hat{V})^2}\right.$$

$$-$$

Variación de

$$\delta\left\{ e^{\frac{2}{(\hat{r}\cdot\hat{V})}}\, \frac{\hat{\mathbf{r}}}{(\hat{r}\cdot\hat{V})} \cdot d\hat{\rho} \right\}$$

$$e^{\frac{2}{(\hat{r}\cdot\hat{V})}} \left[\begin{aligned} & \left[-2\left\{\hat{r}\cdot\hat{A}-1\right\} \frac{\hat{r}\cdot\delta\hat{\rho}}{(\hat{r}\cdot\hat{V})^3} - 2\,\frac{\hat{V}\cdot\delta\hat{\rho}}{(\hat{r}\cdot\hat{V})^2} \right] \frac{\hat{r}\cdot\hat{v}}{\hat{r}\cdot\hat{V}}\, ds \\ & + \left[\frac{\hat{v}\cdot\delta\hat{\rho}}{(\hat{r}\cdot\hat{V})} - \frac{\hat{r}\cdot\delta\hat{\rho}}{(\hat{r}\cdot\hat{V})^2}(\hat{V}\cdot\hat{v}) - \left\{\hat{r}\cdot\hat{A}-1\right\} \frac{\hat{r}\cdot\delta\hat{\rho}}{(\hat{r}\cdot\hat{V})^3}(\hat{r}\cdot\hat{v}) \right] \\ & + \frac{\hat{r}}{\hat{r}\cdot\hat{V}} \cdot d(\delta\hat{\rho}) \end{aligned} \right.$$

Acción adicional J^*

$$\boxed{J^* = -\int_a^b e^{\frac{2}{(\hat{r}\cdot\hat{V})}} \frac{\hat{r}}{(\hat{r}\cdot\hat{V})} \cdot d\hat{\varrho}}$$

$$- \frac{\hat{V}\cdot \delta\hat{\varrho}}{(\hat{r}\cdot\hat{V})^2}(\hat{r}\cdot\hat{v})\Big] ds$$

pag 11 — $\delta\left\{e^{\frac{2}{\hat r\cdot\hat V}}\left[2\hat V - \frac{\hat v}{\hat V\cdot\hat v}\right]\cdot d\hat\varrho\right\}$

$$e^{\frac{2}{\hat r\cdot\hat V}}\left[\begin{array}{l}\left[\begin{array}{l} 2\frac{\hat A\cdot\hat v}{\hat r\cdot\hat V}\hat r + \frac{(\hat A\cdot\hat v)}{(\hat r\cdot\hat V)(\hat V\cdot\hat v)}\hat r + \left\{-4(\hat V\cdot\hat v) + 2\frac{1}{(\hat V\cdot\hat v)}\right\} \\ + \frac{\hat r\cdot\hat v}{\hat r\cdot\hat V}\left\{2\langle(\hat r\cdot\hat A)-1\rangle\frac{\hat r}{(\hat r\cdot\hat V)^3} + 2\frac{\hat V}{(\hat r\cdot\hat V)^2}\right\} \\ - \frac{\hat v}{\hat r\cdot\hat V} + \frac{\hat V\cdot\hat v}{(\hat r\cdot\hat V)^2}\hat r + \langle(\hat r\cdot\hat A)-1\rangle\frac{\hat r\cdot\hat v}{(\hat r\cdot\hat V)^3}\hat r \end{array}\right] \\ + \left[\begin{array}{l}\left\{2\hat V - \frac{2\hat v}{(\hat V\cdot\hat v)} + \frac{\hat V}{(\hat V\cdot\hat v)^2}\right\} \\ - \frac{\hat r}{\hat r\cdot\hat V}\end{array}\right] d\,\delta\hat\varrho \end{array}\right.$$

$$+\delta\left\{e^{\frac{2}{\hat{r}\cdot\hat{V}}}\frac{\hat{r}}{\hat{r}\cdot\hat{V}}\,d\hat{\varrho}\right\}$$ pag 26

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$$\left[\left\{\left(\left(\hat{r}\cdot\hat{A}\right)-1\right)\frac{\hat{r}}{(\hat{r}\cdot\hat{V})^3}+\frac{-\hat{V}}{(\hat{r}\cdot\hat{V})^2}\right\}+\frac{\hat{r}\cdot\hat{v}}{(\hat{r}\cdot\hat{V})^2}\,\hat{V}\right]\cdot\delta\hat{\varrho}\,ds$$

$$
\begin{aligned}
\delta\left\{ e^{\frac{2}{\hat{r}\cdot\hat{V}}}\,\frac{\hat{r}}{(\hat{r}\cdot\hat{V})}\cdot d\hat{\rho}\right\} = e^{\frac{2}{\hat{r}\cdot\hat{V}}}\Bigg[& -\frac{2(\hat{r}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})}\left[\left\{\hat{r}\cdot\hat{A}-1\right\}\frac{\hat{r}}{(\hat{r}\cdot\hat{V})^{3}}\right. \\
& +\left[\frac{\hat{v}}{(\hat{r}\cdot\hat{V})}-\frac{(\hat{V}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})^{2}}\,\hat{r}\right]\cdot \\
& \cancel{\frac{\hat{r}\cdot\hat{v}}{\hat{r}\cdot\hat{V}}}-(\hat{r}\cdot\hat{v})\left[\left\{\hat{r}\cdot\hat{A}-1\right\}\right. \\
& +\frac{\hat{r}}{\hat{r}\cdot\hat{V}}\cdot d(\delta\hat{\rho})
\end{aligned}
$$

Comprobación de $\delta\left\{e^{\frac{2}{(\hat{r}\cdot\hat{V})}}\frac{\hat{r}}{(\hat{r}\cdot\hat{V})}\cdot d\hat{\varrho}\right\}$

1) $\boxed{\delta\hat{r}=\delta\hat{\varrho}-\frac{\hat{r}\cdot\delta\hat{\varrho}}{(\hat{r}\cdot\hat{V})}\hat{V}}$ pág. 6.

2) $\boxed{\delta\frac{1}{(\hat{r}\cdot\hat{V})}=-\left\{\hat{r}\cdot\hat{A}-1\right\}\frac{\hat{r}\cdot\delta\hat{\varrho}}{(\hat{r}\cdot\hat{V})^{3}}-\frac{\hat{V}\cdot\delta\hat{\varrho}}{(\hat{r}\cdot\hat{V})^{2}}}$ pág. 8.

$+\left.\frac{\hat{V}}{(\hat{r}\cdot\hat{V})^{2}}\right]\cdot\delta\hat{\varrho}\,ds$

$\delta\hat{\varrho}\,ds$

$\left.\frac{\hat{r}}{(\hat{r}\cdot\hat{V})^{3}}+\frac{\hat{V}}{(\hat{r}\cdot\hat{V})^{2}}\right]\cdot\delta\hat{\varrho}\,ds$

$$\delta\left[e^{\frac{2}{\hat{r}\cdot\hat{V}}}\frac{\hat{r}}{(\hat{r}\cdot\hat{V})}\cdot d\hat{\rho}\right]=e^{\frac{2}{\hat{r}\cdot\hat{V}}}\left[\left\{\begin{array}{l} -\,2\dfrac{(\hat{r}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})^{4}}\{\hat{r}\cdot\hat{A} \\ \\ -\dfrac{(\hat{r}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})^{3}}\{\hat{r}\cdot\hat{A}- \\ \\ \end{array}\right. \;+\;\frac{\hat{r}}{(\hat{r}\cdot\hat{V})}\cdot d(\delta\hat{\rho})\right]$$

¡Comprobado!

$$-1\}\,\hat{r} - 2\,\frac{(\hat{r}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})^3}\hat{V} + \frac{1}{\hat{r}\cdot\hat{V}}\hat{v} - \frac{\hat{V}\cdot\hat{v}}{(\hat{r}\cdot\hat{V})^2}\hat{r}$$

$$1\}\,\hat{r} - \frac{(\hat{r}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})^2}\hat{V} \quad \cdot\, \delta\hat{z}\,ds$$

$$\delta\left[e^{\frac{2}{\hat{r}\cdot\hat{V}}}\left\{2\hat{V}-\frac{\hat{v}}{\hat{V}\cdot\hat{v}}\right.\right.$$

$$e^{\frac{2}{\hat{r}\cdot\hat{V}}}\left[\left[2\frac{\hat{A}\cdot\hat{v}}{\hat{r}\cdot\hat{V}}\hat{r}+\frac{(\hat{A}\cdot\hat{v})\,(\cancel{\hat{V}\cdot d\hat{s}})}{(\hat{r}\cdot\hat{V})(\hat{V}\cdot\hat{v})^2}\hat{r}-2\left\{2\hat{V}\cdot\hat{v}\right.\right.\right.$$

$$-4(\hat{V}\cdot\hat{v})(\hat{r}\cdot$$

$$+2\frac{(\hat{r}\cdot\hat{A})}{(\hat{V}\cdot\hat{v})}$$

$$-\frac{4(\hat{V}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})^2}$$

$$- \frac{\hat{r}}{\hat{r}\cdot\hat{V}} \Big\} \cdot d\hat{\rho} \Big]$$

$$- \frac{1}{\hat{V}\cdot\hat{v}} \Big\} \Big\{ \langle \hat{r}\cdot\hat{A} - 1 \rangle \frac{\hat{r}}{(\hat{r}\cdot\hat{V})^3} + \frac{\hat{V}}{(\hat{r}\cdot\hat{V})^2} \Big\} \Big] \cdot \delta\hat{\rho}\, ds$$

$$\hat{A}) \frac{\hat{r}}{(\hat{r}\cdot\hat{V})^3} + 4 (\hat{V}\cdot\hat{v}) \frac{\hat{r}}{(\hat{r}\cdot\hat{V})^3}$$

$$(\hat{r}\cdot\hat{V})^3\, \hat{r} - \frac{2}{(\hat{V}\cdot\hat{v})(\hat{r}\cdot\hat{V})^3}\, \hat{r}$$

$$\hat{V} + \frac{2}{(\hat{V}\cdot\hat{v})(\hat{r}\cdot\hat{V})^2}\, \hat{V}$$

$$\delta\left[e^{\frac{2}{\hat{r}\cdot\hat{V}}}\left\{2\hat{V}-\frac{\hat{v}}{(\hat{V}\cdot\hat{v})}-\right.\right.$$

$$e^{\frac{2}{(\hat{r}\cdot\hat{V})}}\Bigg[\Bigg[\frac{2(\hat{A}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})}\hat{r}+\frac{(\hat{A}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})(\hat{V}\cdot\hat{v})^2}\hat{r}-\frac{4(\hat{V}\cdot\hat{v})(\hat{r}\cdot\hat{A})}{(\hat{r}\cdot\hat{V})^3}$$

$$-\frac{4(\hat{V}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})^2}\hat{V}+\frac{2}{(\hat{V}\cdot\hat{v})(\hat{r}\cdot\hat{V})^2}\hat{V}+2(\hat{r}\cdot\hat{v})$$

$$+\frac{(\hat{V}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})^2}\hat{r}+\frac{(\hat{r}\cdot\hat{v})(\hat{r}\cdot\hat{A})}{(\hat{r}\cdot\hat{V})^3}\hat{r}-\frac{(\hat{r}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})^3}\hat{r}\Bigg]$$

$$+\Bigg[2\hat{V}-\frac{2\hat{v}}{(\hat{V}\cdot\hat{v})}+\frac{\hat{V}}{(\hat{V}\cdot\hat{v})^2}-\frac{\hat{r}}{(\hat{r}\cdot\hat{V})}\Bigg]\Bigg]$$

$$\frac{\hat{r}}{(\hat{r}\cdot\hat{V})}\Big\}\cdot d\hat{\varrho}\Big]$$

$$\hat{r} + \frac{4(\hat{V}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})^3}\hat{r} + \frac{2(\hat{r}\cdot\hat{A})}{(\hat{V}\cdot\hat{v})(\hat{r}\cdot\hat{V})^3}\hat{r} - \frac{2}{(\hat{V}\cdot\hat{v})(\hat{r}\cdot\hat{V})^3}\hat{r}$$

$$\frac{(\hat{r}\cdot\hat{A})}{(\hat{r}\cdot\hat{V})^4}\hat{r} - \frac{2(\hat{r}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})^4}\hat{r} + 2\frac{(\hat{r}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})^3}\hat{V} - \frac{1}{(\hat{r}\cdot\hat{V})}\hat{v}\Bigg]\cdot\delta\hat{\varrho}\,ds$$

menos menos

$$+\frac{(\hat{r}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})^2}\hat{V}$$

$$\Big]\cdot d(\delta\hat{\varrho})$$

Campo Central

Consideraciones preliminares:

$$\hat{V} = (1, 0, 0, 0)$$

$$\hat{v} = \left(\frac{1}{\sqrt{1-v^2}}, \frac{v^x}{\sqrt{1-v^2}}, \frac{v^y}{\sqrt{1-v^2}}, \frac{v^z}{\sqrt{1-v^2}}\right)$$

Para el campo central.

$$\hat{V} \cdot \hat{v} = \frac{1}{\sqrt{1-v^2}}.$$

$$(\hat{V} \cdot \hat{v})^2 = \frac{1}{1-v^2}$$

$$1 - v^2 = \frac{1}{(\hat{V} \cdot \hat{v})^2} \qquad \sqrt{1-v^2} = \frac{1}{(\hat{V} \cdot \hat{v})}$$

$$v^2 = 1 - \frac{1}{(\hat{V} \cdot \hat{v})^2}$$

$$\boxed{1 + v^2 = 2 - \frac{1}{(\hat{V} \cdot \hat{v})^2}}$$

$$d\hat{r} = (dt, dx, dy, dz)$$

$$d\hat{r} = [1, v^x, v^y, v^z]\, dt$$

$$\hat{v} \cdot d\hat{r} = \sqrt{1-v^2}\, dt$$

$$dt = \frac{\hat{v} \cdot d\hat{r}}{\sqrt{1-v^2}} \qquad \boxed{dt = (\hat{V} \cdot \hat{v})(\hat{v} \cdot d\hat{r})}$$

$$\boxed{\hat{V}\cdot d\hat{r} = dt}$$

~~Luego~~ Integrales de acción posibles:

$$\boxed{\int e^{\frac{2}{\hat{r}\cdot\hat{V}}}\left[2-\frac{1}{(\hat{V}\cdot\hat{v})^2}\right](\hat{V}\cdot\hat{v})(\hat{v}\cdot d\hat{r})}$$

$$\boxed{\int e^{\frac{2}{\hat{r}\cdot\hat{V}}}\left[2-\frac{1}{(\hat{V}\cdot\hat{v})^2}\right]\hat{V}\cdot d\hat{r}}$$

$$d\hat{r} = d\hat{\varrho} - d\hat{R}$$

$$\boxed{d\hat{r} = d\hat{\varrho} - \hat{V}\,dS}$$

$$(t-T)^2-(x-X)^2-(y-Y)^2-(z-Z)^2=0$$

$$(t-T)(dt-dT)-(x-X)(dx-dX)-(y-Y)(dy-dY)\ldots=0$$

$$\hat{r}\cdot d\hat{r}=0$$

$$\hat{r}\cdot(d\hat{\varrho}-\hat{V}\,dS)=0$$

$$\hat{r}\cdot d\hat{\varrho}-(\hat{r}\cdot\hat{V})\,dS=0$$

$$\boxed{dS = \frac{\hat{r} \cdot d\hat{\rho}}{\hat{r} \cdot \hat{V}}}$$

$$\boxed{d\hat{r} = d\hat{\rho} - \hat{V}\frac{\hat{r} \cdot d\hat{\rho}}{(\hat{r} \cdot \hat{V})}}$$

$$\hat{V} \cdot d\hat{r} = \hat{V} \cdot d\hat{\rho} - \frac{\hat{r} \cdot d\hat{\rho}}{(\hat{r} \cdot \hat{V})}$$

$$\boxed{\hat{V} \cdot d\hat{r} = \left[\hat{V} - \frac{1}{(\hat{r} \cdot \hat{V})}\hat{r}\right] \cdot d\hat{\rho}}$$

$$\boxed{J_1 = \int e^{\frac{2}{\hat{r} \cdot \hat{V}}}\left[2 - \frac{1}{(\hat{V} \cdot \hat{V})^2}\right]\left[\hat{V} - \frac{1}{\hat{r} \cdot \hat{V}}\hat{r}\right] \cdot d\hat{\rho}}$$

$$\boxed{J_1 = \int e^{\frac{2}{\hat{r} \cdot \hat{V}}}\left[2\hat{V} - \frac{2}{\hat{r} \cdot \hat{V}}\hat{r} - \frac{1}{(\hat{V} \cdot \hat{V})^2}\hat{V} + \frac{1}{(\hat{r} \cdot \hat{V})(\hat{V} \cdot \hat{V})^2}\hat{r}\right] \cdot d\hat{\rho}}$$

$$\boxed{J_1 = \int e^{\frac{2}{\hat{r}\cdot\hat{V}}}\left[2\hat{V} - \frac{1}{(\hat{V}\cdot\hat{v})^2}\hat{V} - \frac{2}{\hat{r}\cdot\hat{V}}\hat{r} + \frac{1}{(\hat{r}\cdot\hat{V})(\hat{V}\cdot\hat{v})^2}\hat{r}\right]\cdot d\hat{\rho}}$$

$$\hat{v}\cdot d\hat{r} = \hat{v}\cdot d\hat{\rho} - \frac{(\hat{V}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})}\hat{r}\cdot d\hat{\rho}$$

$$\boxed{\hat{v}\cdot d\hat{r} = \left[\hat{v} - \frac{(\hat{V}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})}\hat{r}\right]\cdot d\hat{\rho}}$$

$$\left[2(\hat{V}\cdot\hat{v}) - \frac{1}{\hat{V}\cdot\hat{v}}\right]\left[\hat{v} - \frac{\hat{V}\cdot\hat{v}}{\hat{r}\cdot\hat{V}}\hat{r}\right]$$

$$\left[2(\hat{V}\cdot\hat{v})\hat{v} - \frac{1}{(\hat{V}\cdot\hat{v})}\hat{v} - \frac{2(\hat{V}\cdot\hat{v})^2}{(\hat{r}\cdot\hat{V})}\hat{r} + \frac{1}{(\hat{r}\cdot\hat{V})}\hat{r}\right]$$

$$\boxed{J_2 = \int e^{\frac{2}{\hat{r}\cdot\hat{V}}}\left[2(\hat{V}\cdot\hat{v})\hat{v} - \frac{1}{(\hat{V}\cdot\hat{v})}\hat{v} - \frac{2(\hat{V}\cdot\hat{v})^2}{(\hat{r}\cdot\hat{V})}\hat{r} + \frac{1}{\hat{r}\cdot\hat{V}}\hat{r}\right]\cdot\delta\hat{\rho}}$$

Comprobación

Campo Central.

$$\hat{V} = (1, 0, 0, 0)$$

$$\hat{v} = \left(\frac{1}{\sqrt{1-v^2}}, \frac{v^x}{\sqrt{1-v^2}}, \frac{v^y}{\sqrt{1-v^2}}, \frac{v^z}{\sqrt{1-v^2}}\right)$$

$$\hat{V}\cdot\hat{v} = \frac{1}{\sqrt{1-v^2}}$$

$$(\hat{V}\cdot\hat{v})^2 = \frac{1}{1-v^2}$$

$$1 - v^2 = \frac{1}{(\hat{V}\cdot\hat{v})^2}$$

$$\boxed{1 + v^2 = 2 - \frac{1}{(\hat{V}\cdot\hat{v})^2}}$$

$$d\hat{r} = (dt, dx, dy, dz)$$

$$\boxed{d\hat{r} = (1, v^x, v^y, v^z)\,dt}$$

$$\boxed{\hat{V}\cdot d\hat{r} = dt}$$

$$\boxed{J_1 = \int e^{\frac{2}{\hat{r}\cdot\hat{V}}}\left[2 - \frac{1}{(\hat{V}\cdot\hat{v})^2}\right](\hat{V}\cdot d\hat{r})}$$

$$\hat{v}\cdot d\hat{r} = \sqrt{1-v^2}\,dt = ds$$

$$dt = \frac{\hat{v}\cdot d\hat{r}}{\sqrt{1-v^2}}$$

$$dt = (\hat{V}\cdot\hat{v})(\hat{v}\cdot d\hat{r})$$

$$\boxed{J_2 = \int e^{\frac{2}{\hat{r}\cdot\hat{V}}}\left[2 - \frac{1}{(\hat{V}\cdot\hat{v})^2}\right](\hat{V}\cdot\hat{v})(\hat{v}\cdot d\hat{r})}$$

Cálculo de $(\hat{V}\cdot d\hat{r})$ y de $(\hat{v}\cdot d\hat{r})$

$$\boxed{d\hat{r} = d\hat{\rho} - d\hat{R}}$$

$$d\hat{r} = d\hat{\rho} - \hat{V}\,dS$$

$$\boxed{\hat{V}\cdot d\hat{r} = \hat{V}\cdot d\hat{\rho} - dS}$$

Cálculo de dS.

$$\hat{r}\cdot d\hat{r} = 0$$

$$\hat{r}\cdot d\hat{\rho} - (\hat{r}\cdot\hat{V})$$

~~Cálculo de dS.~~ ~~(Ver pág 11')~~

$$\hat{r}\cdot\hat{r} = 0 \qquad \hat{r}\cdot d\hat{r} = 0$$

$$\hat{r}\cdot(\hat{\rho} - \hat{R}) = 0$$

~~Calcúlense los diferenciales de ambos miembros de~~

$$\hat{r}\cdot(\hat{v}ds - \hat{V}dS)$$

Cálculo de dS

$$\hat{r}\cdot\hat{r} = 0 \qquad \hat{r} = \hat{\rho} - \hat{R}$$

$$\hat{r}^2 = 0 \qquad d\hat{r} = d\hat{\rho} - d\hat{R}$$

$$\hat{r}\cdot d\hat{r} = 0 \leftarrow d\hat{r} = \hat{v}ds - \hat{V}dS$$

$$\hat{r}\cdot(\hat{v}ds - \hat{V}dS) = 0$$

$$(\hat{r}\cdot\hat{v})\,ds - (\hat{r}\cdot\hat{V})\,dS = 0$$

$$dS = \frac{(\hat{r}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})}\,ds$$

$$dS = \frac{\hat{r}\cdot d\hat{\rho}}{\hat{r}\cdot\hat{V}} = \frac{1}{(\hat{r}\cdot\hat{V})}\,\hat{r}\cdot d\hat{\rho}$$

$$\boxed{dS = \frac{1}{\hat{r}\cdot\hat{V}}\,\hat{r}\cdot d\hat{\rho}}$$

$$\boxed{J_1 = \int e^{\frac{2}{\hat{r}\cdot\hat{V}}}\left[2 - \frac{1}{(\hat{V}\cdot\hat{v})^2}\right]\frac{1}{\hat{r}\cdot\hat{V}}\,\hat{r}\cdot d\hat{\varsigma}}$$

Para el campo central

$$\hat{V} = (1, 0, 0, 0)$$

$$\hat{v} = \left(\frac{1}{\sqrt{1-v^2}}, \frac{v^x}{\sqrt{1-v^2}}, \frac{v^y}{\sqrt{1-v^2}}, \frac{v^z}{\sqrt{1-v^2}}\right)$$

$$\hat{V}\cdot\hat{v} = \frac{1}{\sqrt{1-v^2}}$$

$$(\hat{V}\cdot\hat{v})^2 = \frac{1}{1-v^2}$$

$$2 - \frac{1}{(\hat{V}\cdot\hat{v})^2} = 2 - 1 + v^2 = 1 + v^2$$

$$\hat{r} = (r, x, y, z)$$

$$\hat{V} = (1, 0, 0, 0)$$

$$\boxed{\hat{r}\cdot\hat{V} = r} \qquad d\hat{\varsigma} = (dt, dx, dy, dz)$$

$$\hat{r}\cdot d\hat{\varsigma} = r\,dt$$

$$\boxed{\frac{\hat{r}\cdot d\hat{\varsigma}}{\hat{r}\cdot\hat{V}} = dt.}$$

$$\boxed{J_1 = \int \frac{e^{\frac{2}{\hat{r}\cdot\hat{V}}}}{\hat{r}\cdot\hat{V}}\left[2 - \frac{1}{(\hat{V}\cdot\hat{V})^2}\right]\hat{r}\cdot d\hat{\varrho}}$$

$$\boxed{J_1 = \int \frac{e^{\frac{2}{\hat{r}\cdot\hat{V}}}}{\hat{r}\cdot\hat{V}}\left[2\hat{r} - \frac{\hat{r}}{(\hat{V}\cdot\hat{V})^2}\right]\cdot d\hat{\varrho}}$$

Cálculo de la variación de S.

$\boxed{\delta S}$ Ver página 5 y 6.

$$(\hat{r})^2 = 0$$

$$\hat{r}\cdot\delta\hat{r} = 0$$

$$\hat{r}\cdot\left[\delta\hat{\varrho} - \hat{V}\,\delta S\right] = 0$$

$$\hat{r}\cdot\delta\hat{\varrho} - \hat{r}\cdot\hat{V}\,\delta S = 0$$

$$\delta S = \frac{\hat{r}}{\hat{r}\cdot\hat{V}}\cdot\delta\hat{\varrho}$$

$$\boxed{\delta S = \frac{1}{\hat{r}\cdot\hat{V}}\,\hat{r}\cdot\delta\hat{\varrho}}$$

págs. 5 y 6

$$\boxed{\delta \hat{r}}$$

$$\hat{r} = \hat{\varrho} - \hat{R}$$

$$\delta \hat{r} = \delta \hat{\varrho} - \delta \hat{R}$$

$$\delta \hat{r} = \delta \hat{\varrho} - \hat{V} \delta \delta$$

$$\boxed{\delta \hat{r} = \delta \hat{\varrho} - \hat{V} \frac{\hat{r} \cdot \delta \hat{\varrho}}{(\hat{r} \cdot \hat{V})}}$$

pág. 6

$$\boxed{\delta \hat{V}}$$

$$J_1 = \int e^{\frac{2}{\hat{r}\cdot\hat{V}}} \left[2 - \frac{1}{(\hat{V}\cdot\hat{v})^2}\right]\left[\hat{V} - \frac{\hat{r}}{\hat{r}\cdot\hat{V}}\right]\cdot d\hat{\varsigma}$$

$$J_1 = \int e^{\frac{2}{\hat{r}\cdot\hat{V}}} \left[2\hat{V} - \frac{2\hat{r}}{\hat{r}\cdot\hat{V}} - \frac{\hat{V}}{(\hat{V}\cdot\hat{v})^2} + \frac{\hat{r}}{(\hat{V}\cdot\hat{v})^2(\hat{r}\cdot\hat{V})}\right]\cdot d\hat{\varsigma}$$

$$\boxed{J_1 = \int e^{\frac{2M}{\hat{r}\cdot\hat{V}}} \left[2\hat{V} - \frac{\hat{V}}{(\hat{V}\cdot\hat{v})^2} - \frac{2\hat{r}}{(\hat{r}\cdot\hat{V})} + \frac{\hat{r}}{(\hat{V}\cdot\hat{v})^2(\hat{r}\cdot\hat{V})}\right]\cdot d\hat{\varsigma}}$$

Calcular las extremales del problema variacional

$$\delta\int_a^b e^{\frac{2M}{\hat r\cdot\hat V}}\left[2\hat V - \frac{\hat V}{(\hat V\cdot\hat v)^2} - \frac{2\hat r}{(\hat r\cdot\hat V)} + \frac{\hat r}{(\hat V\cdot\hat v)^2(\hat r\cdot\hat V)}\right]\cdot d\hat\rho = 0$$

Programa. Soluciones página 70.

1) ¡ $\delta\hat\rho$ es libre.!

2) δS;

3) $\delta\hat V$;

4) $\delta\hat r$;

5) $\delta(ds)$;

6) $\delta\hat v$;

7) $\delta(\hat r\cdot\hat V)$;

8) $\delta\frac{1}{(\hat r\cdot\hat V)}$;

9) $(\hat V\cdot\hat v)$;

10) $\delta\frac{1}{(\hat V\cdot\hat v)^2}$;

11) $\delta\frac{\hat V}{(\hat V\cdot\hat v)^2}$;

12) $\delta\frac{\hat r}{(\hat r\cdot\hat V)}$;

13) $\delta\left[\frac{1}{(\hat V\cdot\hat v)^2}\frac{\hat r}{(\hat r\cdot\hat V)}\right]$;

14) $\delta\left[2\hat V - \frac{\hat V}{(\hat V\cdot\hat v)^2} - \frac{2\hat r}{\hat r\cdot\hat V} + \frac{\hat r}{(\hat V\cdot\hat v)^2(\hat r\cdot\hat V)}\right]$;

15) $\delta\left(\frac{2M}{\hat r\cdot\hat V}\right)$

2) Cálculo de δS.

$$\hat{r}\cdot\hat{r} = 0\ ;$$

$$\hat{r}\cdot\delta\hat{r} = 0\ ;$$

$$\hat{r}\cdot\delta[\hat{\varrho} - \hat{R}] = 0\ ;$$

$$\hat{r}\cdot[\delta\hat{\varrho} - \hat{V}\delta S] = 0$$

$$\hat{r}\cdot\delta\hat{\varrho} - \hat{r}\cdot\hat{V}\delta S = 0$$

2) $$\boxed{\delta S = \frac{\hat{r}}{\hat{r}\cdot\hat{V}}\cdot\delta\hat{\varrho}}$$

Ver página 6.

3) Cálculo de $\delta\hat{V}$.

$$\delta\hat{V} = \hat{A}\,\delta S$$

$$\delta\hat{V} = \hat{A}\,\frac{\hat{r}\cdot\delta\hat{\varrho}}{(\hat{r}\cdot\hat{V})}$$

3) $$\boxed{\delta\hat{V} = \hat{A}\,\frac{(\hat{r}\cdot\delta\hat{\varrho})}{(\hat{r}\cdot\hat{V})}}$$

Ver página 7.

4) Cálculo de $\delta\hat{r}$.

$$\hat{r} = \hat{\rho} - \hat{R}.$$

$$\delta\hat{r} = \delta\hat{\rho} - \hat{V}\,\delta S$$

4) $$\boxed{\delta\hat{r} = \delta\hat{\rho} - \hat{V}\,\frac{(\hat{r}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})}}$$ Ver página 6.

5) Cálculo de ~~δs~~. $\delta(ds)$

$$ds = \sqrt{d\hat{\rho}\cdot d\hat{\rho}}$$

$$\delta(ds) = \frac{d\hat{\rho}\cdot d(\delta\hat{\rho})}{\sqrt{d\hat{\rho}\cdot d\hat{\rho}}} = \frac{d\hat{\rho}\cdot d(\delta\hat{\rho})}{ds}$$

5) $$\boxed{\delta\, ds = \hat{v}\cdot d(\delta\hat{\rho})}$$ Ver página 5.

6) Cálculo de $\delta\hat{v}$.

$$\hat{v} = \frac{d\hat{\rho}}{ds}$$

$$\delta\hat{v} = \frac{ds\, d(\delta\hat{\rho}) - [\hat{v}\cdot d(\delta\hat{\rho})]\, d\hat{\rho}}{ds^2}$$

$$6) \quad \boxed{\delta\hat{v} = \frac{d}{ds}\delta\hat{\rho} - \hat{v}\left[\hat{v}\cdot\frac{d}{ds}(\delta\hat{\rho})\right]}$$

Ver página 7.

7) Cálculo de $\delta(\hat{r}\cdot\hat{V})$.

$$\delta(\hat{r}\cdot\hat{V}) = \hat{V}\cdot\delta\hat{r} + \hat{r}\cdot\delta\hat{V}$$

$$7) \quad \boxed{\delta(\hat{r}\cdot\hat{V}) = \hat{V}\cdot\delta\hat{\rho} - \frac{\hat{r}\cdot\delta\hat{\rho}}{\hat{r}\cdot\hat{V}} + \frac{(\hat{r}\cdot\hat{A})}{(\hat{r}\cdot\hat{V})}(\hat{r}\cdot\delta\hat{\rho})}$$

Ver página 7.

8) Cálculo de $\delta\left(\frac{1}{\hat{r}\cdot\hat{V}}\right)$

menos

$$8) \quad \boxed{\delta\frac{1}{(\hat{r}\cdot\hat{V})} = -\frac{\hat{V}\cdot\delta\hat{\rho}}{(\hat{r}\cdot\hat{V})^2} + \frac{\hat{r}\cdot\delta\hat{\rho}}{(\hat{r}\cdot\hat{V})^3} - \frac{(\hat{r}\cdot\hat{A})}{(\hat{r}\cdot\hat{V})^3}(\hat{r}\cdot\delta\hat{\rho})}$$

Ver página 8.

9) Cálculo de $\delta(\hat{V}\cdot\hat{v})$.

$$\delta(\hat{V}\cdot\hat{v}) = \hat{V}\cdot\delta\hat{v} + \hat{v}\cdot\delta\hat{V}.$$

$$\delta(\hat{V}\cdot\hat{v}) = \left[\hat{V}\cdot\frac{d}{ds}(\delta\hat{\rho}) - (\hat{V}\cdot\hat{v})\left[\hat{v}\cdot\frac{d}{ds}(\delta\hat{\rho})\right] + (\hat{v}\cdot\hat{A})\frac{\hat{r}\cdot\delta\hat{\rho}}{(\hat{r}\cdot\hat{V})}\right]$$

9)
$$\boxed{\delta(\hat{V}\cdot\hat{v}) = \left[(\hat{v}\cdot\hat{A})\frac{\hat{r}\cdot\delta\hat{\rho}}{\hat{r}\cdot\hat{V}} + \hat{V}\cdot\frac{d}{ds}(\delta\hat{\rho}) - (\hat{V}\cdot\hat{v})\left[\hat{v}\cdot\frac{d}{ds}(\delta\hat{\rho})\right]\right]}$$

Ver página 8.

10) Cálculo de $\delta\frac{1}{(\hat{V}\cdot\hat{v})^2}$.

menos

10)
$$\delta\frac{1}{(\hat{V}\cdot\hat{v})^2} = \left[-\frac{2(\hat{v}\cdot\hat{A})}{(\hat{V}\cdot\hat{v})^3}\frac{(\hat{r}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})} - \frac{2\hat{V}\cdot\frac{d}{ds}(\delta\hat{\rho})}{(\hat{V}\cdot\hat{v})^3} + \frac{2\left[\hat{v}\cdot\frac{d}{ds}(\delta\hat{\rho})\right]}{(\hat{V}\cdot\hat{v})^2}\right]$$

11) Cálculo de $\delta \frac{\hat{V}}{(\hat{V}\cdot\hat{v})^2}$.

$$\delta \frac{\hat{V}}{(\hat{V}\cdot\hat{v})^2} = \left[\frac{(\hat{r}\cdot\delta\hat{\varrho})}{(\hat{r}\cdot\hat{V})(\hat{V}\cdot\hat{v})^2}\hat{A} - \frac{2(\hat{v}\cdot\hat{A})(\hat{r}\cdot\delta\hat{\varrho})}{(\hat{r}\cdot\hat{V})(\hat{V}\cdot\hat{v})^3}\hat{V} - \frac{2\left[\hat{V}\cdot\frac{d}{ds}(\delta\hat{\varrho})\right]}{(\hat{V}\cdot\hat{v})^3}\hat{V} + \frac{2\left[\hat{v}\cdot\frac{d}{ds}(\delta\hat{\varrho})\right]}{(\hat{V}\cdot\hat{v})^2}\hat{V} \right]$$

12) Cálculo de $\delta \frac{\hat{r}}{(\hat{r}\cdot\hat{V})}$.

$$\delta\left(\frac{\hat{r}}{\hat{r}\cdot\hat{V}}\right) = \left[-\frac{(\hat{V}\cdot\delta\hat{\varrho})}{(\hat{r}\cdot\hat{V})^2}\hat{r} + \frac{\hat{r}\cdot\delta\hat{\varrho}}{(\hat{r}\cdot\hat{V})^3}\hat{r} - \frac{(\hat{r}\cdot\hat{A})(\hat{r}\cdot\delta\hat{\varrho})}{(\hat{r}\cdot\hat{V})^3}\hat{r} + \frac{1}{(\hat{r}\cdot\hat{V})}\delta\hat{\varrho} - \frac{\hat{r}\cdot\delta\hat{\varrho}}{(\hat{r}\cdot\hat{V})^2}\hat{V} \right]$$

13) Cálculo de $\delta\left[\frac{1}{(\hat{V}\cdot\hat{v})^2}\frac{\hat{r}}{(\hat{r}\cdot\hat{V})}\right]$.

$$\delta\left[\frac{1}{(\hat{V}\cdot\hat{v})^2}\frac{\hat{r}}{(\hat{r}\cdot\hat{V})}\right] = \Bigg[-\frac{(\hat{V}\cdot\delta\hat{\varrho})}{(\hat{r}\cdot\hat{V})^2(\hat{V}\cdot\hat{v})^2}\hat{r} + \frac{(\hat{r}\cdot\delta\hat{\varrho})}{(\hat{r}\cdot\hat{V})^3(\hat{V}\cdot\hat{v})^2}\hat{r} - \frac{(\hat{r}\cdot\hat{A})(\hat{r}\cdot\delta\hat{\varrho})}{(\hat{r}\cdot\hat{V})^3(\hat{V}\cdot\hat{v})^2}\hat{r}$$

$$+ \frac{1}{(\hat{r}\cdot\hat{V})(\hat{V}\cdot\hat{v})^2}\delta\hat{\varrho} - \frac{(\hat{r}\cdot\delta\hat{\varrho})}{(\hat{r}\cdot\hat{V})^2(\hat{V}\cdot\hat{v})^2}\hat{V}$$

$$- \frac{2(\hat{v}\cdot\hat{A})(\hat{r}\cdot\delta\hat{\varrho})}{(\hat{r}\cdot\hat{V})^2(\hat{V}\cdot\hat{v})^3}\hat{r} - \frac{2\left[\hat{V}\cdot\frac{d}{ds}(\delta\hat{\varrho})\right]}{(\hat{r}\cdot\hat{V})(\hat{V}\cdot\hat{v})^3}\hat{r}$$

$$+ \frac{2\left[\hat{v}\cdot\frac{d}{ds}(\delta\hat{\varrho})\right]}{(\hat{r}\cdot\hat{V})(\hat{V}\cdot\hat{v})^2}\hat{r}\Bigg]$$

14) Cálculo de

$$\delta\left[2\hat{V} - \frac{\hat{V}}{(\hat{V}\cdot\hat{v})^2} - \frac{2\hat{r}}{(\hat{r}\cdot\hat{V})} + \frac{\hat{r}}{(\hat{r}\cdot\hat{V})(\hat{V}\cdot\hat{v})^2}\right]$$

$$2\frac{(\hat{r}\cdot\delta\hat{\rho})}{\hat{r}\cdot\hat{V}}\hat{A} - \frac{(\hat{r}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})(\hat{V}\cdot\hat{v})^2}\hat{A} + \frac{2(\hat{v}\cdot\hat{A})(\hat{r}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})(\hat{V}\cdot\hat{v})^3}$$

$$+\frac{2(\hat{V}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})^2}\hat{r} - \frac{2(\hat{r}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})^3}\hat{r} + \frac{2(\hat{r}\cdot\hat{A})(\hat{r}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})^3}$$

$$-\frac{(\hat{V}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})^2(\hat{V}\cdot\hat{v})^2}\hat{r} + \frac{(\hat{r}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})^3(\hat{V}\cdot\hat{v})^2}\hat{r} - \frac{(\hat{r}\cdot\hat{A})(\hat{r}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})^3(\hat{V}\cdot\hat{v})}$$

$$-\frac{2(\hat{v}\cdot\hat{A})(\hat{r}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})^2(\hat{V}\cdot\hat{v})^3}\hat{r} - \frac{2\left[\hat{V}\cdot\frac{d}{ds}(\delta\hat{\rho})\right]}{(\hat{r}\cdot\hat{V})(\hat{V}\cdot\hat{v})^3}\hat{r}$$

$$\hat{V} + \frac{2\left[\hat{V}\cdot\frac{d}{ds}(\delta\hat{\rho})\right]}{(\hat{V}\cdot\hat{v})^3}\hat{V} - \frac{2\left[\hat{v}\cdot\frac{d}{ds}(\delta\hat{\rho})\right]}{(\hat{V}\cdot\hat{v})^2}\hat{V}$$

$$\hat{r} - \frac{2}{(\hat{r}\cdot\hat{V})}\delta\hat{\rho} + \frac{2(\hat{r}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})^2}\hat{V}$$

$$= \hat{r} + \frac{1}{(\hat{r}\cdot\hat{V})(\hat{V}\cdot\hat{v})^2}\delta\hat{\rho} - \frac{(\hat{r}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})^2(\hat{V}\cdot\hat{v})^2}\hat{V}$$

$$+ \frac{2\left[\hat{v}\cdot\frac{d}{ds}(\delta\hat{\rho})\right]}{(\hat{r}\cdot\hat{V})(\hat{V}\cdot\hat{v})^2}\hat{r}$$

$$\boxed{\left[2\hat{V} - \frac{\hat{V}}{(\hat{V}\cdot\hat{v})^2} - \frac{2\hat{r}}{(\hat{r}\cdot\hat{V})} + \frac{\hat{r}}{(\hat{r}\cdot\hat{V})(\hat{V}\cdot\hat{v})^2}\right]\cdot d\hat{s}}$$

15) Cálculo de $\delta \dfrac{2M}{(\hat{r}\cdot\hat{V})}$

$$\boxed{\delta \frac{2M}{(\hat{r}\cdot\hat{V})} = -\frac{2M(\hat{V}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})^2} + \frac{2M(\hat{r}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})^3} - \frac{2M(\hat{r}\cdot\hat{A})(\hat{r}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})^3}}$$

$$\boxed{= \left[2(\hat{V}\cdot\hat{v}) - \frac{(\hat{V}\cdot\hat{v})}{(\hat{V}\cdot\hat{v})^2} - \frac{2(\hat{r}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})} + \frac{(\hat{r}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})(\hat{V}\cdot\hat{v})^2} \right] ds}$$

Principio Variacional.

$M_{ij} = M_{ji}$ Tensor

$N_i = \hat{N}$ Cuadrivector

$$\delta \int_A^B \frac{M_{ij}\, dx^i dx^j}{N_p\, dx^p} = 0$$

Numerador: ν

Denominador: $d\tau$

$$\delta \nu = \left\{ \begin{array}{l} \frac{\partial M_{ij}}{\partial x^k} dx^i dx^j \delta x^k + M_{ij} \cdot d(\delta x^i) dx^j \\ + M_{ij}\, dx^i\, d(\delta x^j) \end{array} \right\}$$

$$\delta(d\tau) = \frac{\partial N_p}{\partial x^q} \delta x^q dx^p + N_p\, d(\delta x^p)$$

$$Q = \frac{\nu}{d\tau} = \text{Cociente.}$$

$$\delta Q = \frac{d\tau\, \delta\nu - \nu\, \delta(d\tau)}{(d\tau)^2}$$

$$\underline{d\tau\,\delta\gamma - \nu\,\delta(d\tau)}$$

$$\left[\begin{aligned}
&N_p \frac{\partial M_{ij}}{\partial x^k}\, dx^i\, dx^j\, dx^p\, \delta x^k + N_p M_{ij}\, dx^p\, dx^j\, d(\delta x^i) \\
&+ N_p M_{ij}\, dx^i\, dx^p\, d(\delta x^j) \\
&- M_{ij} \frac{\partial N_p}{\partial x^q}\, dx^i\, dx^j\, dx^p\, \delta x^q \\
&- M_{ij} N_p\, dx^i\, dx^j\, d(\delta x^p)
\end{aligned}\right]$$

$$\delta Q = \left[\begin{aligned}
&N_p \frac{\partial M_{ij}}{\partial x^k} \frac{dx^i}{d\tau} \frac{dx^j}{d\tau} \frac{dx^p}{d\tau}\, \delta x^k\, d\tau \\
&+ N_p M_{kj} \frac{dx^p}{d\tau} \frac{dx^j}{d\tau}\, d(\delta x^k) \\
&+ N_p M_{ik} \frac{dx^i}{d\tau} \frac{dx^p}{d\tau}\, d(\delta x^k) \\
&- M_{ij} \frac{\partial N_p}{\partial x^k} \frac{dx^i}{d\tau} \frac{dx^j}{d\tau} \frac{dx^p}{d\tau}\, \delta x^k \\
&- M_{ij} N_k \frac{dx^i}{d\tau} \frac{dx^j}{d\tau}\, d(\delta x^k)
\end{aligned}\right]$$

$$\int_A^B \delta Q$$

$$\boxed{\int_A^B \delta Q}$$

$$
\begin{aligned}
\int_A^B \Bigg[& N_p \frac{\partial M_{ij}}{\partial x^k} \frac{dx^i}{d\tau} \frac{dx^j}{d\tau} \frac{dx^p}{d\tau} \\
& - M_{ij} \frac{\partial N_p}{\partial x^k} \frac{dx^i}{d\tau} \frac{dx^j}{d\tau} \frac{dx^p}{d\tau} \\
& + \frac{\partial N_p}{\partial x^i} M_{kj} \frac{dx^i}{d\tau} \frac{dx^j}{d\tau} \frac{dx^p}{d\tau} \\
& - N_p \frac{\partial M_{kj}}{\partial x^i} \frac{dx^i}{d\tau} \frac{dx^j}{d\tau} \frac{dx^p}{d\tau} \\
& - N_p M_{kj} \frac{d^2 x^p}{d\tau^2} \frac{dx^j}{d\tau} \\
& - N_p M_{kj} \frac{dx^p}{d\tau} \frac{d^2 x^j}{d\tau^2} \\
& - \frac{\partial N_p}{\partial x^j} M_{ik} \frac{dx^i}{d\tau} \frac{dx^j}{d\tau} \frac{dx^p}{d\tau} \\
& - N_p \frac{\partial M_{ik}}{\partial x^j} \frac{dx^i}{d\tau} \frac{dx^j}{d\tau} \frac{dx^p}{d\tau} \\
& - N_p M_{ik} \frac{d^2 x^i}{d\tau^2} \frac{dx^p}{d\tau} \\
& - N_p M_{ik} \frac{dx^i}{d\tau} \frac{d^2 x^p}{d\tau^2} \\
& + \frac{\partial M_{ij}}{\partial x^p} N_k \frac{dx^i}{d\tau} \frac{dx^j}{d\tau} \frac{dx^p}{d\tau} \\
& + M_{ij} \frac{\partial N_k}{\partial x^p} \frac{dx^i}{d\tau} \frac{dx^j}{d\tau} \frac{dx^p}{d\tau} \\
& + M_{ij} N_k \frac{d^2 x^i}{d\tau^2} \frac{dx^j}{d\tau} + M_{ij} N_k \frac{dx^i}{d\tau} \frac{d^2 x^j}{d\tau^2} \Bigg] \delta x^k \, d\tau
\end{aligned}
$$

$$\cancel{\frac{d^2x^r}{ds^2}} \quad \frac{dx^r}{ds} = \frac{dx^r}{d\tau}\frac{d\tau}{ds}$$

$$\frac{d^2x^r}{ds^2} = \frac{d^2x^r}{d\tau^2}\left(\frac{d\tau}{ds}\right)^2 + \frac{dx^r}{d\tau}\frac{d^2\tau}{ds^2}$$

$$d\tau = N_p\, dx^p$$

$$\frac{d\tau}{ds} = N_p \frac{dx^p}{ds}$$

$$\frac{d^2\tau}{ds^2} = \frac{\partial N_p}{\partial x^q}\frac{dx^p}{ds}\frac{dx^q}{ds} + N_p \frac{d^2x^p}{ds^2}$$

$$\frac{d^2\tau}{ds^2} = \frac{\partial N_p}{\partial x^q}\frac{dx^p}{ds}\frac{dx^q}{ds} + N_p\left[\frac{\partial h_{pm}}{\partial x^n} - \frac{\partial h_{mn}}{\partial x^p}\right]\frac{dx^m}{ds}\frac{dx^n}{ds}$$

$$\frac{d^2x^r}{d\tau^2} = \frac{\frac{d^2x^r}{ds^2} - \frac{dx^r}{d\tau}\frac{d^2\tau}{ds^2}}{\left(\frac{d\tau}{ds}\right)^2}$$

$$\frac{d^2x^r}{d\tau^2} = \frac{\left[\frac{\partial h_{rm}}{\partial x^n} - \frac{\partial h_{mn}}{\partial x^r}\right]\frac{dx^m}{ds}\frac{dx^n}{ds} - \frac{dx^r}{d\tau}\frac{d^2\tau}{ds^2}}{\left(\frac{d\tau}{ds}\right)^2}$$

$$\boxed{d\tau = N_p \, dx^p}$$

$$\boxed{\frac{d\tau}{ds} = N_p \frac{dx^p}{ds}}$$

$$\frac{d^2\tau}{ds^2} = \frac{\partial N_p}{\partial x^q}\frac{dx^p}{ds}\frac{dx^q}{ds} + N_p \frac{d^2x^p}{ds^2}$$

$$\frac{d^2\tau}{ds^2} = \frac{\partial N_p}{\partial x^q}\frac{dx^p}{ds}\frac{dx^q}{ds} + N_p\left[\frac{\partial h_{pm}}{\partial x^n} - \frac{\partial h_{mn}}{\partial x^p}\right]\frac{dx^m}{ds}\frac{dx^n}{ds}$$

$$\boxed{\frac{d^2\tau}{ds^2} = \left[\frac{\partial N_m}{\partial x^n} + N_p\frac{\partial h_{pm}}{\partial x^n} - N_p\frac{\partial h_{mn}}{\partial x^p}\right]\frac{dx^m}{ds}\frac{dx^n}{ds}}$$

$$\cancel{\frac{dx^i}{d\tau} = \frac{dx^i}{ds}\frac{d\tau}{d}}$$

$$\boxed{\frac{dx^i}{ds} = \frac{dx^i}{d\tau}\frac{d\tau}{ds}}$$

$$\frac{d^2x^i}{ds^2} = \frac{d^2x^i}{d\tau^2}\left(\frac{d\tau}{ds}\right)^2 + \frac{dx^i}{d\tau}\frac{d^2\tau}{ds^2}$$

$$\boxed{\frac{d^2x^i}{d\tau^2} = \frac{\frac{d^2x^i}{ds^2} - \frac{dx^i}{d\tau}\frac{d^2\tau}{ds^2}}{\left(\frac{d\tau}{ds}\right)^2}}$$

$$\frac{d^2x^i}{d\tau^2} = \left[\frac{\partial h_{im}}{\partial x^n} - \frac{\partial h_{mn}}{\partial x^i}\right]\frac{dx^m}{d\tau}\frac{dx^n}{d\tau} - \left[\frac{\partial N_m}{\partial x^n} + N_p\frac{\partial h_{pm}}{\partial x^n} - N_p\frac{\partial h_{mn}}{\partial x^p}\right]\frac{dx^i}{d\tau}\frac{dx^m}{d\tau}\frac{dx^n}{d\tau}$$

$$\boxed{\frac{d^2x^i}{d\tau^2} = \left[\frac{\partial h_{im}}{\partial x^n} - \frac{\partial h_{mn}}{\partial x^i}\right]\frac{dx^m}{d\tau}\frac{dx^n}{d\tau} + \left[N_p\frac{\partial h_{mn}}{\partial x^p} - N_p\frac{\partial h_{pm}}{\partial x^n} - \frac{\partial N_m}{\partial x^n}\right]\frac{dx^i}{d\tau}\frac{dx^m}{d\tau}\frac{dx^n}{d\tau}}$$

$$(1+v^2)\,dt$$

$$\hat{V} = (1, 0, 0, 0)$$

$$\hat{v} = \left(\frac{1}{\sqrt{1-v^2}}, \frac{v^x}{\sqrt{1-v^2}}, \frac{v^y}{\sqrt{1-v^2}}, \frac{v^z}{\sqrt{1-v^2}}\right)$$

$$\hat{V}\cdot\hat{v} = \frac{1}{\sqrt{1-v^2}}$$

$$\frac{1}{(\hat{V}\cdot\hat{v})^2} = 1 - v^2$$

$$\boxed{1 - \frac{1}{(\hat{V}\cdot\hat{v})^2} = v^2}$$

$$\boxed{1 + v^2 = 2 + \frac{1}{(\hat{V}\cdot\hat{v})^2}}$$

$$dt = d\hat{r}\cdot\hat{V}$$

$$d\hat{r} = d\hat{\varrho} - d\hat{R}$$

$$\cancel{d\hat{R} = d\hat{\varrho}}$$

$$\boxed{d\hat{r} = d\hat{\varrho} - \hat{V}\,\frac{\hat{r}\cdot d\hat{\varrho}}{\hat{r}\cdot\hat{V}}}$$

$$\boxed{dt = \hat{V}\cdot d\hat{\varrho} - \frac{\hat{r}\cdot d\hat{\varrho}}{\hat{r}\cdot\hat{V}}}$$

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$$(1+v^2)\,dt = \left[2 \mp \frac{1}{(\hat{V}\cdot\hat{v})^2}\right]\left[\hat{V} - \frac{\hat{r}}{(\hat{r}\cdot\hat{V})}\right]\cdot d\hat{\rho}$$

$$2\hat{V}\cdot d\hat{\rho} - \frac{\hat{v}\cdot d\hat{\rho}}{(\hat{V}\cdot\hat{v})}$$

$$\hat{V} = (1, 0, 0, 0)$$

$$d\hat{\rho} = (dt, dx, dy, dz)$$

$$\hat{V}\cdot d\hat{\rho} = dt$$

$$\cancel{\hat{V}\cdot d\hat{\rho} = (\hat{V}\cdot\hat{v})\,ds} \qquad \boxed{\hat{V}\cdot\hat{v} = \frac{1}{\sqrt{1-v^2}}}$$

$$\hat{v} = \left(\frac{1}{\sqrt{1-v^2}}, \frac{v^x}{\sqrt{1-v^2}}, \frac{v^y}{\sqrt{1-v^2}}, \frac{v^z}{\sqrt{1-v^2}}\right)$$

$$\hat{v}\cdot d\hat{\rho} = \frac{dt - v^x dx - v^y dy - v^z dz}{\sqrt{1-v^2}}$$

$$2\,dt - \frac{dt - v^x dx - v^y dy - v^z dz}{\sqrt{1-v^2}}$$

$$2\,dt - \frac{dt^2 - dx^2 - dy^2 - dz^2}{dt\sqrt{1-v^2}}$$

$$2\,dt - ds^2$$

$$\hat{v} \cdot d\hat{\rho} = \frac{dt^2 - dx^2 - dy^2 - dz^2}{dt\sqrt{1-v^2}}$$

$$\frac{\hat{v} \cdot d\hat{\rho}}{(\hat{V} \cdot \hat{v})} = \frac{dt^2 - dx^2 - dy^2 dz^2}{dt}$$

$$2\hat{V} \cdot d\hat{\rho} = 2dt$$

$$2\hat{V} \cdot d\hat{\rho} - \frac{\hat{v} \cdot d\hat{\rho}}{\hat{V} \cdot \hat{v}} = \frac{2dt^2 - dt^2 + dx^2 + dy^2 + dz^2}{dt}$$

$$= \cancel{dt}\,(1 + v^2)\,dt$$

$$\hat{V} \cdot d\hat{\rho} = dt$$

$$\left[2 - \frac{1}{(\hat{V} \cdot \hat{v})^2}\right] (\hat{V} \cdot d\hat{\rho})$$

$$-\frac{\hat{V} \cdot d\hat{\rho}}{(\hat{V} \cdot \hat{v})^2} \qquad - \frac{\hat{v} \cdot d\hat{\rho}}{\hat{V} \cdot \hat{v}}$$

$$\frac{(\hat{V} \cdot \hat{v})\, ds}{(\hat{V} \cdot \hat{v})^2} = \frac{ds}{(\hat{V} \cdot \hat{v})} \qquad - \frac{ds}{(\hat{V} \cdot \hat{v})}$$

$$2)\quad \boxed{\delta S = \frac{\hat{r}}{\hat{r}\cdot\hat{V}}\cdot\delta\hat{\rho}}$$

$$3)\quad \boxed{\delta\hat{V} = \frac{(\hat{r}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})}\hat{A}}$$

$$4)\quad \boxed{\delta\hat{r} = \delta\hat{\rho} - \frac{(\hat{r}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})}\hat{V}}$$

$$5)\quad \boxed{\delta(ds) = \hat{v}\cdot d(\delta\hat{\rho})}$$

$$6)\quad \boxed{\delta\hat{v} = \frac{d}{ds}(\delta\hat{\rho}) - \hat{v}\left\{\hat{v}\cdot\frac{d}{ds}\delta\hat{\rho}\right\}}$$

$$7)\quad \boxed{\delta(\hat{r}\cdot\hat{V}) = \hat{V}\cdot\delta\hat{\rho} - \frac{\hat{r}\cdot\delta\hat{\rho}}{\hat{r}\cdot\hat{V}} + \frac{\hat{r}\cdot\hat{A}}{\hat{r}\cdot\hat{V}}(\hat{r}\cdot\delta\hat{\rho})}$$

$$8)\quad \boxed{\delta\frac{1}{(\hat{r}\cdot\hat{V})} = -\frac{\hat{V}\cdot\delta\hat{\rho}}{(\hat{r}\cdot\hat{V})^2} + \frac{\hat{r}\cdot\delta\hat{\rho}}{(\hat{r}\cdot\hat{V})^3} - \frac{\hat{r}\cdot\hat{A}}{(\hat{r}\cdot\hat{V})^3}(\hat{r}\cdot\delta\hat{\rho})}$$

$$9)\quad \boxed{\delta(\hat{V}\cdot\hat{v}) = (\hat{v}\cdot\hat{A})\frac{\hat{r}\cdot\delta\hat{\rho}}{\hat{r}\cdot\hat{V}} + \hat{V}\cdot\frac{d}{ds}(\delta\hat{\rho}) - (\hat{V}\cdot\hat{v})\left[\hat{v}\cdot\frac{d}{ds}(\delta\hat{\rho})\right]}$$

$$10)\quad \boxed{\delta\frac{1}{(\hat{V}\cdot\hat{v})^2} = -\frac{2(\hat{v}\cdot\hat{A})}{(\hat{V}\cdot\hat{v})^3}\frac{(\hat{r}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})} - \frac{2\left[\hat{V}\cdot\frac{d}{ds}(\delta\hat{\rho})\right]}{(\hat{V}\cdot\hat{v})^3} + \frac{2\left[\hat{v}\cdot\frac{d}{ds}(\delta\hat{\rho})\right]}{(\hat{V}\cdot\hat{v})^2}}$$

$$11)\quad \delta \frac{\hat{V}}{(\hat{V}\cdot\hat{v})^2} = \left[\begin{aligned} &\frac{(\hat{r}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})(\hat{V}\cdot\hat{v})^2}\hat{A} - \frac{2(\hat{v}\cdot\hat{A})(\hat{r}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})(\hat{V}\cdot\hat{v})^3}\hat{V} \\ &- \frac{2\left[\hat{V}\cdot\frac{d}{ds}(\delta\hat{\rho})\right]}{(\hat{V}\cdot\hat{v})^3}\hat{V} + \frac{2\left[\hat{v}\cdot\frac{d}{ds}(\delta\hat{\rho})\right]}{(\hat{V}\cdot\hat{v})^2}\hat{V}. \end{aligned} \right]$$

$$12)\quad \delta \frac{\hat{r}}{(\hat{r}\cdot\hat{V})} = \left[\begin{aligned} &-\frac{(\hat{V}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})^2}\hat{r} + \frac{(\hat{r}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})^3}\hat{r} - \frac{(\hat{r}\cdot\hat{A})(\hat{r}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})^3}\hat{r} \\ &+ \frac{1}{(\hat{r}\cdot\hat{V})}\delta\hat{\rho} - \frac{(\hat{r}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})^2}\hat{V} \end{aligned} \right]$$

$$13)\quad \delta\left[\frac{1}{(\hat{V}\cdot\hat{v})^2}\,\frac{\hat{r}}{(\hat{r}\cdot\hat{V})}\right] = \left[\begin{aligned} &-\frac{(\hat{V}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})^2(\hat{V}\cdot\hat{v})^2}\hat{r} + \frac{(\hat{r}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})^3(\hat{V}\cdot\hat{v})^2}\hat{r} \\ &-\frac{(\hat{r}\cdot\hat{A})(\hat{r}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})^3(\hat{V}\cdot\hat{v})^2}\hat{r} + \frac{1}{(\hat{r}\cdot\hat{V})(\hat{V}\cdot\hat{v})^2}\delta\hat{\rho} \\ &-\frac{(\hat{r}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})^2(\hat{V}\cdot\hat{v})^2}\hat{V} - \frac{2(\hat{v}\cdot\hat{A})(\hat{r}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})^2(\hat{V}\cdot\hat{v})^3}\hat{r} \\ &-\frac{2\left[\hat{V}\cdot\frac{d}{ds}(\delta\hat{\rho})\right]}{(\hat{r}\cdot\hat{V})(\hat{V}\cdot\hat{v})^3}\hat{r} + \frac{2\left[\hat{v}\cdot\frac{d}{ds}(\delta\hat{\rho})\right]}{(\hat{r}\cdot\hat{V})(\hat{V}\cdot\hat{v})^2}\hat{r} \end{aligned} \right]$$

$$14)\quad \delta\left[2\hat{V} - \frac{\hat{V}}{(\hat{V}\cdot\hat{v})^2} - \frac{2\hat{r}}{(\hat{r}\cdot\hat{V})} + \frac{\hat{r}}{(\hat{r}\cdot\hat{V})(\hat{V}\cdot\hat{v})^2}\right]$$

$$2\frac{(\hat{r}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})}\hat{A} - \frac{(\hat{r}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})(\hat{V}\cdot\hat{v})^2}\hat{A} + \frac{2(\hat{v}\cdot\hat{A})(\hat{r}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})(\hat{V}\cdot\hat{v})^3}\hat{V} \quad + \frac{2(\hat{V}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})^2}$$

$$-\frac{2}{\hat{r}\cdot\hat{V}}\delta\hat{\rho} + \frac{2(\hat{r}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})^2}\hat{V} - \frac{(\hat{V}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})^2(\hat{V}\cdot\hat{v})^2}\hat{r} \quad + \frac{(\hat{r}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})^3(\hat{V}\cdot}$$

$$+\frac{1}{(\hat{r}\cdot\hat{V})(\hat{V}\cdot\hat{v})^2}\delta\hat{\rho} - \frac{(\hat{r}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})^2(\hat{V}\cdot\hat{v})^2}\hat{V} - \frac{2(\hat{v}\cdot\hat{A})(\hat{r}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})(\hat{V}\cdot\hat{v})^3}\hat{r}$$

$$+\frac{2\left[\hat{V}\cdot\frac{d}{ds}(\delta\hat{\rho})\right]}{(\hat{V}\cdot\hat{v})^3}\hat{V} - \frac{2\left[\hat{v}\cdot\frac{d}{ds}(\delta\hat{\rho})\right]}{(\hat{V}\cdot\hat{v})^2}\hat{V} \quad - \frac{2[\hat{V}\cdot}{(\hat{r}\cdot\hat{V})(}$$

$$15)\quad \delta\frac{2M}{(\hat{r}\cdot\hat{V})}$$

$$16)\quad \left[2\hat{V} - \frac{\hat{V}}{(\hat{V}\cdot\hat{v})^2} - \frac{2\hat{r}}{(\hat{r}\cdot\hat{V})} + \frac{\hat{r}}{(\hat{r}\cdot\hat{V})(\hat{V}\cdot\hat{v})}\right]\cdot d\hat{\rho} = \left[2\,(\hat{V}\cdot\hat{v}) - \frac{1}{(\hat{V}\cdot}\right.$$

$$+\frac{\hat{r}}{(\hat{r}\cdot\hat{V})(\hat{V}\cdot\hat{v})^2}\Big]$$

$$\frac{(\hat{r}\cdot\delta\hat{\rho})}{(\hat{V}\cdot\hat{v})^3}\hat{V} + \frac{2(\hat{V}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})^2}\hat{r} - \frac{2(\hat{r}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})^3}\hat{r} + \frac{2(\hat{r}\cdot\hat{A})(\hat{r}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})^3}\hat{r}$$

$$\frac{}{(\hat{V}\cdot\hat{v})^2}\hat{r} + \frac{(\hat{r}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})^3(\hat{V}\cdot\hat{v})^2}\hat{r} - \frac{(\hat{r}\cdot\hat{A})(\hat{r}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})^3(\hat{V}\cdot\hat{v})^2}\hat{r}$$

$$\hat{V} - \frac{2(\hat{V}\cdot\hat{A})}{(\hat{r}\cdot\hat{V})^2}\frac{(\hat{r}\cdot\delta\hat{\rho})}{(\hat{V}\cdot\hat{v})^3}\hat{r}$$

$$\delta\hat{\rho})\Big]\hat{V} - \frac{2\left[\hat{V}\cdot\frac{d}{ds}(\delta\hat{\rho})\right]}{(\hat{r}\cdot\hat{V})(\hat{V}\cdot\hat{v})^3}\hat{r} + \frac{2\left[\hat{v}\cdot\frac{d}{ds}(\delta\hat{\rho})\right]}{(\hat{r}\cdot\hat{V})(\hat{V}\cdot\hat{v})^2}\hat{r}$$

15) $\delta\frac{2M}{(\hat{r}\cdot\hat{V})} = \Big[-\frac{2M(\hat{V}\cdot\delta\hat{\rho})}{} + \frac{2M(\hat{r}\cdot\delta\hat{\rho})}{}\Big]$

Considérese la <u>diferencial</u>

$$d\left[e^{\frac{2M}{(\hat{r}\cdot\hat{V})}} f(\hat{r}\cdot\hat{V})\right].$$

f es una función que se especificará más tarde.

$$d\left[e^{\frac{2M}{(\hat{r}\cdot\hat{V})}} f(\hat{r}\cdot\hat{V})\right] =$$

$$e^{\frac{2M}{\hat{r}\cdot\hat{V}}}\left[\begin{array}{l} f(\hat{r}\cdot\hat{V})\left[\begin{array}{l} -2M\{\hat{r}\cdot\hat{A}-1\}\dfrac{\hat{r}\cdot d\hat{\rho}}{(\hat{r}\cdot\hat{V})^3} \\ -2M\dfrac{\hat{V}\cdot d\hat{\rho}}{(\hat{r}\cdot\hat{V})^2} \end{array}\right] \\ + f'(\hat{r}\cdot\hat{V})\left[\{\hat{r}\cdot\hat{A}-1\}\dfrac{\hat{r}\cdot d\hat{\rho}}{\hat{r}\cdot\hat{V}} + \hat{V}\cdot d\hat{\rho}\right] \end{array}\right]$$

$$d\left[e^{\frac{2M}{(\hat{r}\cdot\hat{\vec{V}})}} f(\hat{r}\cdot\hat{\vec{V}})\right] =$$

$$e^{\frac{2M}{(\hat{r}\cdot\hat{\vec{V}})}}\left[(\hat{r}\cdot\hat{A})\left\langle -2M f(\hat{r}\cdot\hat{\vec{V}})\frac{\hat{r}\cdot d\hat{\rho}}{(\hat{r}\cdot\hat{\vec{V}})^3} + f'(\hat{r}\cdot\hat{V})\frac{\hat{r}\cdot d\hat{\rho}}{\hat{r}\cdot\hat{\vec{V}}} \right\rangle + \left\langle +2M f(\hat{r}\cdot\hat{\vec{V}})\frac{\hat{r}\cdot d\hat{\rho}}{(\hat{r}\cdot\hat{\vec{V}})^3} - 2M f(\hat{r}\cdot\hat{\vec{V}})\frac{\hat{\vec{V}}\cdot d\hat{\rho}}{(\hat{r}\cdot\hat{\vec{V}})^2} - f'(\hat{r}\cdot\hat{\vec{V}})\frac{\hat{r}\cdot d\hat{\rho}}{\hat{r}\cdot\hat{\vec{V}}} + f'(\hat{r}\cdot\hat{\vec{V}})\,\hat{\vec{V}}\cdot d\hat{\rho} \right\rangle \right]$$

$$d\left[e^{\frac{2M}{(\hat{r}\cdot\hat{V})}} f(\hat{r}\cdot\hat{V})\right] =$$

$$(\hat{r}\cdot\hat{A})\, e^{\frac{2M}{\hat{r}\cdot\hat{V}}} \left\langle \left\{ \frac{-2Mf}{(\hat{r}\cdot\hat{V})^3} + \frac{f'}{\hat{r}\cdot\hat{V}} \right\} \hat{r} \right\rangle \cdot d\hat{\rho}$$

$$+\, e^{\frac{2M}{\hat{r}\cdot\hat{V}}} \left\langle \left\{ \frac{+2Mf}{(\hat{r}\cdot\hat{V})^3} - \frac{f'}{\hat{r}\cdot\hat{V}} \right\} \hat{r}\cdot d\hat{\rho} + \left\{ -\frac{2Mf}{(\hat{r}\cdot\hat{V})^2} + f' \right\} \hat{V}\cdot d\hat{\rho} \right\rangle$$

$$d\left[e^{\frac{2M}{\hat r\cdot\hat V}} f(\hat r\cdot\hat V)\right] =$$

$$(\hat r\cdot\hat A)\, e^{\frac{2M}{\hat r\cdot\hat V}}\left\{\frac{-2Mf}{(\hat r\cdot\hat V)^2}+f'\right\}\frac{\hat r}{(\hat r\cdot\hat V)}\cdot d\hat\rho$$

$$+\, e^{\frac{2M}{\hat r\cdot\hat V}}\left(-\left\{-\frac{2Mf}{(\hat r\cdot\hat V)^2}+f'\right\}\frac{\hat r}{\hat r\cdot\hat V}\cdot d\hat\rho \atop +\left\{-\frac{2Mf}{(\hat r\cdot\hat V)^2}+f'\right\}\hat V\cdot d\hat\rho\right)$$

comprobado. ~~¡Falta comprobar!~~

Lema: Cuando $\hat A = 0$

$\Longrightarrow$

$$e^{\frac{2M}{(\hat r\cdot\hat V)}}\left(-\left\{-\frac{2Mf}{(\hat r\cdot\hat V)^2}+f'\right\}\frac{\hat r}{\hat r\cdot\hat V}\cdot d\hat\rho \atop +\left\{-\frac{2Mf}{(\hat r\cdot\hat V)^2}+f'\right\}\hat V\cdot d\hat\rho\right)$$

¡es una diferencial exacta!

$$\boxed{e^{\frac{2M}{(\hat{r}\cdot\hat{V})}}\left\{\frac{2Mf}{(\hat{r}\cdot\hat{V})^2}-f'\right\}\left\{\frac{\hat{r}}{\hat{r}\cdot\hat{V}}-\hat{V}\right\}\cdot d\hat{\rho}}$$

¡comprobado! ~~¡falta comprobar!~~

Esta expresión es una diferencial exacta cuando $\hat{A}=0$.

$$\boxed{f=f(\hat{r}\cdot\hat{V})}$$

Principio variacional

$$-J = \int_A^B \frac{m}{2}\, e^{\frac{2M}{(\hat{V}\cdot\hat{r})}} \left(2 - \frac{1}{(\hat{V}\cdot\hat{v})^2}\right)(\hat{V}\cdot d\hat{\rho})$$

$$\delta e^{\frac{2M}{(\hat{V}\cdot\hat{r})}} = e^{\frac{2M}{(\hat{V}\cdot\hat{r})}} \left[-\frac{2M(\hat{r}\cdot\hat{A})(\hat{r}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})^3} + \frac{2M(\hat{r}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})^3} - \frac{2M(\hat{V}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})^2} \right]$$

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$$\delta \frac{1}{(\hat{V}\cdot\hat{v})^2} = -\frac{2}{(\hat{V}\cdot\hat{v})^3} \left[\frac{(\hat{A}\cdot\hat{v})(\hat{r}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})} + \hat{V}\cdot\left\{\frac{d}{ds}(\delta\hat{\rho})\right\} - \left\{\hat{v}\cdot\left\{\frac{d}{ds}(\delta\hat{\rho})\right\}\right\}(\hat{V}\cdot\hat{v}) \right]$$

$$\delta\left(2 - \frac{1}{(\hat{V}\cdot\hat{v})^2}\right) = \left[+ \frac{2(\hat{A}\cdot\hat{v})(\hat{r}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})(\hat{V}\cdot\hat{v})^3} + \frac{2\left\langle\hat{V}\cdot\left\{\frac{d}{ds}(\delta\hat{\rho})\right\}\right\rangle}{(\hat{V}\cdot\hat{v})^3} - \frac{2\left\langle\hat{v}\cdot\left\{\frac{d}{ds}(\delta\hat{\rho})\right\}\right\rangle\xi}{(\hat{V}\cdot\hat{v})^2} \right.$$

$$\boxed{\delta(\hat{V}\cdot d\hat{\rho}) = \frac{(\hat{r}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})}(\hat{A}\cdot d\hat{\rho}) + \hat{V}\cdot\{d(\delta\hat{\rho})\}}$$

$$\delta\left\langle e^{\frac{2M}{(\hat{V}\cdot\hat{r})}}\left[2-\frac{1}{(\hat{V}\cdot\hat{v})^2}\right](\hat{V}\cdot d\hat{\rho})\right\rangle$$

$$e^{\frac{2M}{(\hat{V}\cdot\hat{r})}}\Bigg[\Bigg\{-\frac{4M(\hat{A}\cdot\hat{r})(\hat{r}\cdot\delta\hat{\rho})(\hat{V}\cdot\hat{v})}{(\hat{V}\cdot\hat{r})^3}+\frac{4M(\hat{r}\cdot\delta\hat{\rho})(\hat{V}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})^3}-\frac{4M(\hat{V}\cdot\delta\hat{\rho})(\hat{V}\cdot\hat{v})}{(\hat{V}\cdot\hat{r})^2}$$

$$+\frac{2M(\hat{A}\cdot\hat{r})(\hat{r}\cdot\delta\hat{\rho})}{(\hat{V}\cdot\hat{r})^3(\hat{V}\cdot\hat{v})}-\frac{2M(\hat{r}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})^3(\hat{V}\cdot\hat{v})}+\frac{2M(\hat{V}\cdot\delta\hat{\rho})}{(\hat{V}\cdot\hat{r})^2(\hat{V}\cdot\hat{v})}$$

$$+\frac{2(\hat{A}\cdot\hat{v})(\hat{r}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})(\hat{V}\cdot\hat{v})^2}+\frac{2\left\langle\hat{V}\cdot\left\{\frac{d}{ds}(\delta\hat{\rho})\right\}\right\rangle}{(\hat{V}\cdot\hat{v})^2}-\frac{2\left\langle\hat{v}\cdot\left\{\frac{d}{ds}(\delta\hat{\rho})\right\}\right\rangle}{(\hat{V}\cdot\hat{v})}$$

$$+\frac{2(\hat{A}\cdot\hat{v})(\hat{r}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})}\cancel{+}-\frac{(\hat{A}\cdot\hat{v})(\hat{r}\cdot\delta\hat{\rho})}{(\hat{r}\cdot\hat{V})(\hat{V}\cdot\hat{v})^2}\Bigg\}ds$$

$$+2\left\langle\hat{V}\cdot\{d(\delta\hat{\rho})\}\right\rangle-\frac{1}{(\hat{V}\cdot\hat{v})^2}\left\langle\hat{V}\cdot\{d(\delta\hat{\rho})\}\right\rangle\Bigg]$$

$$\delta\left\langle e^{\frac{2M}{(\hat{V}\cdot\hat{r})}}\left[2-\frac{1}{(\hat{V}\cdot\hat{v})^2}\right](\hat{V}\cdot d\hat{\rho})\right\rangle$$

$$e^{\frac{2M}{(\hat{V}\cdot\hat{r})}}\Bigg[\Bigg(-\frac{4M(\hat{A}\cdot\hat{r})(\hat{V}\cdot\hat{v})}{(\hat{V}\cdot\hat{r})^3}\hat{r}+\frac{4M(\hat{V}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})^3}\hat{r}-\frac{4M(\hat{V}\cdot\hat{v})}{(\hat{V}\cdot\hat{r})^2}\hat{V}+\frac{2M(\hat{A}\cdot\hat{r})}{(\hat{V}\cdot\hat{r})^3(\hat{V}\cdot\hat{v})}\hat{r}$$

$$-\frac{2\cdot M}{(\hat{V}\cdot\hat{r})^3(\hat{V}\cdot\hat{v})}\hat{r}+\frac{2M}{(\hat{V}\cdot\hat{r})^2(\hat{V}\cdot\hat{v})}\hat{V}+\frac{2(\hat{A}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})(\hat{V}\cdot\hat{v})^2}\hat{r}+\frac{2(\hat{A}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})}\hat{r}$$

$$-\frac{(\hat{A}\cdot\hat{v})}{(\hat{r}\cdot\hat{V})(\hat{V}\cdot\hat{v})^2}\hat{r}\Bigg)\cdot\delta\hat{\rho}\,ds$$

$$+\left(\frac{2}{(\hat{V}\cdot\hat{v})^2}\hat{V}-\frac{2}{(\hat{V}\cdot\hat{v})}\hat{v}+2\hat{V}-\frac{1}{(\hat{V}\cdot\hat{v})^2}\hat{V}\right)\cdot d(\delta\hat{\rho})\Bigg]$$

$$\delta\left\langle \left[e^{\frac{2M}{(\hat{V}\cdot\hat{r})}} \left\{ 2 - \frac{1}{(\hat{V}\cdot\hat{v})^2} \right\} \right] (\hat{V}\cdot d\hat{\rho}) \right\rangle$$

$$e^{\frac{2M}{(\hat{V}\cdot\hat{r})}} \Bigg[\left(-\frac{4M(\hat{A}\cdot\hat{r})(\hat{V}\cdot\hat{v})}{(\hat{V}\cdot\hat{r})^3}\hat{r} + \frac{4M(\hat{V}\cdot\hat{v})}{(\hat{V}\cdot\hat{r})^3}\hat{r} + \frac{2M(\hat{A}\cdot\hat{r})}{(\hat{V}\cdot\hat{r})^3(\hat{V}\cdot\hat{v})}\hat{r} - \frac{2M}{(\hat{V}\cdot\hat{r})^3(\hat{V}\cdot\hat{v})}\hat{r} + \frac{(\hat{A}\cdot\hat{v})}{(\hat{V}\cdot\hat{r})(\hat{V}\cdot\hat{v})^2}\hat{r} + \frac{2(\hat{A}\cdot\hat{v})}{(\hat{V}\cdot\hat{r})}\hat{r} - \frac{4M(\hat{V}\cdot\hat{v})}{(\hat{V}\cdot\hat{r})^2}\hat{V} + \frac{2M}{(\hat{V}\cdot\hat{r})^2(\hat{V}\cdot\hat{v})}\hat{V} \right) \cdot \delta\hat{\rho}\, ds$$

$$+ \left(+ \frac{1}{(\hat{V}\cdot\hat{v})^2}\hat{V} + 2\hat{V} - \frac{2}{(\hat{V}\cdot\hat{v})}\hat{v} \right) \cdot d(\delta\hat{\rho}) \Bigg]$$

Comprobado

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$$\bar{x}^i = l^i_j x^j$$

$$\Delta_{mn}\bar{x}^m \bar{x}^n = \Delta_{mn} l^m_i x^i l^n_j x^j$$

$$= \Delta_{mn} l^m_i l^n_j x^i x^j = \Delta_{ij} x^i x^j$$

$$\boxed{\Delta_{mn} l^m_i l^n_j = \Delta_{ij}}$$

$$\bar{x} = \frac{x - Vt}{\sqrt{1-V^2}} \quad \bar{t} = \frac{t - Vx}{\sqrt{1-V^2}} \quad \bar{y} = y \quad \bar{z} = z$$

$$\bar{x}^1 = \frac{1}{\sqrt{1-V^2}} x^1 - \frac{V}{\sqrt{1-V^2}} x^2 + 0 x^3 + 0 x^4$$

$$\bar{x}^2 = -\frac{V}{\sqrt{1-V^2}} x^1 + \frac{1}{\sqrt{1-V^2}} x^2 + 0 x^3 + 0 x^4$$

$$\bar{x}^3 = 0 x^1 + 0 x^2 + 1 x^3 + 0 x^4$$

$$\bar{x}^4 = 0 x^1 + 0 x^2 + 0 \cdot x^3 + 1 x^4$$

$$\begin{pmatrix} \frac{1}{\sqrt{1-V^2}} & -\frac{V}{\sqrt{1-V^2}} & 0 & 0 \\ -\frac{V}{\sqrt{1-V^2}} & \frac{1}{\sqrt{1-V^2}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

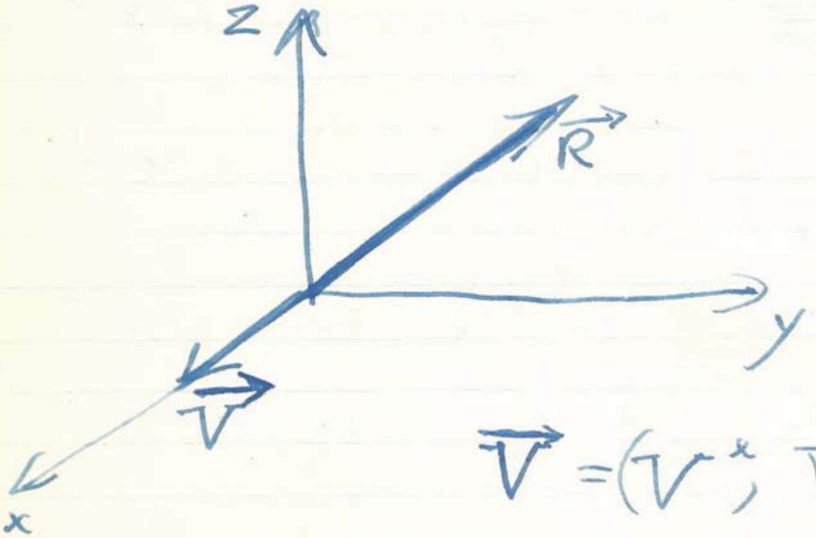
$\frac{1 - V^2}{1 - V^2}$ = producto escalar del renglón 1 por el renglón 1.

$\rho_1 \cdot \rho_1 = 1 \quad \rho_1 \cdot \rho_2 = 0 \quad \rho_1 \cdot \rho_3 = 0 \quad \rho_1 \rho_4 = 0$

$\rho_2 \cdot \rho_2 = -1 \quad \rho_2 \rho_3 = 0 \quad \rho_2 \cdot \rho_4 = 0$

$\rho_3 \rho_3 = -1 \quad \rho_3 \rho_4 = 0 \quad \rho_4 \rho_4 = -1$

$$\bar{x} = \frac{x - Vt}{\sqrt{1-V^2}} \qquad \bar{t} = \frac{t - Vx}{\sqrt{1-V^2}} \qquad \bar{y} = y \qquad \bar{z} = z$$



$$\vec{V} = (V^x, V^y, V^z)$$

$$\bar{x} = x + \left\{-1 + \frac{1}{\sqrt{1-V^2}}\right\} x - \frac{V}{\sqrt{1-V^2}} t$$

$$\bar{x} = x + \{-1 + \beta\} x - \frac{V}{\sqrt{1-V^2}} t$$

$$\bar{y} = y$$

$$\bar{z} = z$$

$$\boxed{\vec{\bar{R}} = \vec{R} + \{-1 + \beta\} \frac{\vec{R}\cdot\vec{V}}{V^2}\vec{V} - \beta \vec{V} t}$$

$$\bar{t} = \frac{1}{\sqrt{1-V^2}} t - \frac{\cancel{V}}{\sqrt{1-V^2}} \cdot \frac{\vec{V}\cdot\vec{R}}{\cancel{V}}$$

$$\boxed{\bar{t} = \frac{1}{\sqrt{1-V^2}} t - \beta \vec{V}\cdot\vec{R}}$$

$$\bar{t} = \frac{1}{\sqrt{1-V^2}}\, t - \beta\left(V^x x + V^y y + V^z z\right)$$

$$\vec{\bar{R}} = \vec{R} + \{-1+\beta\}\, \frac{V^x x + V^y y + V^z z}{V^2}\, \vec{V} - \beta \vec{V} t$$

$$\bar{t} = \beta\left[t - V^x x - V^y y - V^z z\right]$$

$$\vec{\bar{R}} = \vec{R} + \{-1+\beta\}\, \frac{V^x x + V^y y + V^z z}{V^2}\, \vec{V} - \beta \vec{V} t$$

$$\bar{x}^1 = \frac{1}{\sqrt{1-V^2}}\, x^1 - \frac{V^x}{\sqrt{1-V^2}}\, x^2 - \frac{V^y}{\sqrt{1-V^2}}\, x^3 - \frac{V^z}{\sqrt{1-V^2}}\, x^4$$

$$\bar{x}^2 = -\frac{V^x}{\sqrt{1-V^2}}\, x^1 + \left[1 + \{-1+\beta\}\frac{V^x V^x}{V^2}\right] x^2 + \{-1+\beta\}\frac{V^x V^y}{V^2}\, x^3 + \{-1+\beta\}\frac{V^x V^z}{V^2}\, x^4$$

$$\bar{x}^3 = -\frac{V^y}{\sqrt{1-V^2}}\, x^1 + \{-1+\beta\}\frac{V^x V^y}{V^2}\, x^2 + \left[1 + \{-1+\beta\}\frac{V^y V^y}{V^2}\right] x^3 + \{-1+\beta\}\frac{V^y V^z}{V^2}\, x^4$$

$$\bar{x}^4 = -\frac{V^z}{\sqrt{1-V^2}}\, x^1 + \{-1+\beta\}\frac{V^x V^z}{V^2}\, x^2 + \{-1+\beta\}\frac{V^y V^z}{V^2}\, x^3 + \left[1 + \{-1+\beta\}\frac{V^z V^z}{V^2}\right] x^4$$

Matriz

$$
\begin{array}{c|c|c|c}
\dfrac{1}{\sqrt{1-\bar{V}^2}} & -\dfrac{\bar{V}^x}{\sqrt{1-\bar{V}^2}} & -\dfrac{\bar{V}^y}{\sqrt{1-\bar{V}^2}} & -\dfrac{\bar{V}^z}{\sqrt{1-\bar{V}^2}} \\
\hline
\dfrac{-\bar{V}^x}{\sqrt{1-\bar{V}^2}} & 1+\{-1+B\}\dfrac{\bar{V}^x\bar{V}^x}{\bar{V}^2} & \{-1+B\}\dfrac{\bar{V}^x\bar{V}^y}{\bar{V}^2} & \{-1+B\}\dfrac{\bar{V}^x\bar{V}^z}{\bar{V}^2} \\
\hline
-\dfrac{\bar{V}^y}{\sqrt{1-\bar{V}^2}} & \{-1+B\}\dfrac{\bar{V}^y\bar{V}^x}{\bar{V}^2} & 1+\{-1+B\}\dfrac{\bar{V}^y\bar{V}^y}{\bar{V}^2} & \{-1+B\}\dfrac{\bar{V}^y\bar{V}^z}{\bar{V}^2} \\
\hline
-\dfrac{\bar{V}^z}{\sqrt{1-\bar{V}^2}} & \{-1+B\}\dfrac{\bar{V}^z\bar{V}^x}{\bar{V}^2} & \{-1+B\}\dfrac{\bar{V}^z\bar{V}^y}{\bar{V}^2} & 1+\{-1+B\}\dfrac{\bar{V}^z\bar{V}^z}{\bar{V}^2}
\end{array}
$$

$$\boxed{B = \dfrac{1}{\sqrt{1-\bar{V}^2}}}$$

$$\Delta_{mn}\, l^{m\cdot}_{i}\, l^{n\cdot}_{j} = \Delta_{ij}$$

$$\frac{-V^{-x}}{1-V^2} + \frac{V^{-x}}{\sqrt{1-V^2}} + \{-1+\beta\}\frac{V^{-x}\cancel{V^2}}{\sqrt{1-V^2}\,\cancel{V^2}} = 0$$

$$\frac{(V^{-x})^2}{1-V^2} - 1 - \{-1+\beta\}^2\left[\frac{(V^{-x})^2(V^{-x})^2}{V^4} + \frac{(V^{-x})^2(V^{-y})^2}{V^4} + \frac{(V^{-x})^2(V^{-z})^2}{V^4}\right]$$

$$-2\{-1+\beta\}\frac{(V^{-x})^2}{V^2}$$

$$\frac{(V^{-x})^2}{V^4}\left[(V^{-x})^2+(V^{-y})^2+(V^{-z})^2\right] = \frac{(V^{-x})^2}{V^2}$$

$$(-1+\beta)^2 = \left[-1+\frac{1}{\sqrt{1-V^2}}\right]^2 = 1-2\beta+\frac{1}{1-V^2} = \frac{2-V^2}{1-V^2} - 2\beta$$

$$\cancel{\frac{V^{-x^2}}{V^2}}\,(V^{-x})^2\left[\frac{1}{1-V^2} - \frac{2-V^2}{V^2(1-V^2)} + \cancel{\frac{2\beta}{V^2}} + \frac{2}{V^2} - \cancel{\frac{2\beta}{V^2}}\right.$$

$$\left. - \frac{\cancel{V^2} - \cancel{2} + \cancel{V^2} + \cancel{2} - \cancel{2V^2}}{V^2(1-V^2)}\right] = 0$$

acontecimiento partícula exploradora $P(\bar{t}, \bar{x}, \bar{y}, \bar{z})$

acontecimiento retardado partícula Generadora del Campo $\Big\}$ $G(\bar{T}, \bar{X}, \bar{Y}, \bar{Z})$

$$(\bar{t}-\bar{T}, \bar{x}-\bar{X}, \bar{y}-\bar{Y}, \bar{z}-\bar{Z})$$

$$(\bar{t}-\bar{T})^2-(\bar{x}-\bar{X})^2-(\bar{y}-\bar{Y})^2-(\bar{z}-\bar{Z})^2=0$$

Traducir el cuadrivector $t-T,\ x-X,\ y-Y,\ z-Z$

Términos en $\mathcal{R}^2 \vec{v}$ en el desarrollo de $\frac{\vec{a}}{M}$ en función de $\vec{r}, \vec{V}, \vec{A}$ y $\vec{v}$.

$+ B(\vec{V}\cdot\vec{A})\, r$ 1 6

$\cancel{- B(\vec{V}\cdot\vec{A})(\vec{r}\cdot\vec{v})}$

$\cancel{- B^{-1}(\vec{V}\cdot\vec{v})}$ 7

$\cancel{+ B(\vec{V}\cdot\vec{A})(\vec{r}\cdot\vec{v})}$

$+ 2B^3 (\vec{V}\cdot\vec{A})(\vec{r}\cdot\vec{v})$

$\cancel{+ B^{-1}(\vec{V}\cdot\vec{v})}$

$- 2B(\vec{V}\cdot\vec{v})$ 8

$-2B(\vec{A}\cdot\vec{v})\, r$

$- 2B^3(\vec{V}\cdot\vec{A})(\vec{V}\cdot\vec{v})\, r$ 9

$- 2B(\vec{A}\cdot\vec{v})(\vec{r}\cdot\vec{v})$.

$- 2B^3(\vec{V}\cdot\vec{A})(\vec{V}\cdot\vec{v})(\vec{r}\cdot\vec{v})$

$+ 2B(\vec{V}\cdot\vec{v})^2$ 10

$+ 4B(\vec{V}\cdot\vec{v})(\vec{A}\cdot\vec{v})\, r$

$+ 2B^3(\vec{V}\cdot\vec{A})(\vec{V}\cdot\vec{v})^2 r$

$- B(\vec{V}\cdot\vec{A})\, r v^2$ 1 11

Términos en $\mathcal{R}^2\vec{v}$ en el desarrollo de $\frac{\vec{a}}{M}$ en función de $\vec{r}$, $\vec{V}$, $\vec{A}$ y $\vec{v}$.

$$+B(\vec{V}\cdot\vec{A})\,r\,b^{-2}$$
$$+2B^3(\vec{V}\cdot\vec{A})(\vec{r}\cdot\vec{v})$$
$$-2B(\vec{V}\cdot\vec{v})$$
$$-2B(\vec{A}\cdot\vec{v})\,r$$
$$-2B^3(\vec{V}\cdot\vec{A})(\vec{V}\cdot\vec{v})\,r$$
$$-2B(\vec{A}\cdot\vec{v})(\vec{r}\cdot\vec{v})$$
$$-2B^3(\vec{V}\cdot\vec{A})(\vec{V}\cdot\vec{v})(\vec{r}\cdot\vec{v})$$
$$+2B(\vec{V}\cdot\vec{v})^2$$
$$+4B(\vec{V}\cdot\vec{v})(\vec{A}\cdot\vec{v})\,r$$
$$+2B^3(\vec{V}\cdot\vec{A})(\vec{V}\cdot\vec{v})^2\,r$$

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Términos en $\mathcal{R}^3\vec{v}$ en el desarrollo de $\frac{\vec{a}}{M}$ en función de $\vec{r}$, $\vec{V}$, $\vec{A}$ y $\vec{v}$.

$-B^{-1}(\vec{V}\cdot\vec{r})$ ☆ ▲ 1

$-B^{-1}(\vec{A}\cdot\vec{r})\,r$ ☆

$-B^{-3}r$ ☆

$+B^{-1}r$ ☆ 6 ▲ 1

$\cancel{+B^{-3}(\vec{r}\cdot\vec{v})}$

$\cancel{+B^{-1}(\vec{A}\cdot\vec{r})(\vec{r}\cdot\vec{v})}$ 7

$\cancel{-B^{-3}(\vec{r}\cdot\vec{v})}$

$+2B^{-1}(\vec{r}\cdot\vec{v})$ ☆

$\cancel{-B^{-1}(\vec{A}\cdot\vec{r})(\vec{r}\cdot\vec{v})}$

$+2B(\vec{A}\cdot\vec{r})(\vec{r}\cdot\vec{v})$ 8 ☆

$-2B(\vec{V}\cdot\vec{r})(\vec{V}\cdot\vec{v})$ ☆ ▲ 2

$-2B(\vec{V}\cdot\vec{v})(\vec{A}\cdot\vec{r})\,r$ ☆

$+2B(\vec{V}\cdot\vec{v})\,r$ ☆ ▲ 2

$-2B^{-1}(\vec{V}\cdot\vec{v})\,r$ ☆ 9

$-2B^{-1}(\vec{V}\cdot\vec{v})(\vec{r}\cdot\vec{v})$ ☆

$-2B(\vec{V}\cdot\vec{v})(\vec{A}\cdot\vec{r})(\vec{r}\cdot\vec{v})$

10

$+2B(\vec{V}\cdot\vec{v})^2(\vec{V}\cdot\vec{r})$ ☆ ▲ 3

$+2B(\vec{V}\cdot\vec{v})^2(\vec{A}\cdot\vec{r})\,r$ ☆

$+2B^{-1}(\vec{V}\cdot\vec{v})^2\,r$ ☆

$-2B(\vec{V}\cdot\vec{v})^2\,r$ ☆ ▲ 3

$+B^{-1}(\vec{V}\cdot\vec{r})\,v^2$ ☆ ▲ 4

$+B^{-1}(\vec{A}\cdot\vec{r})\,r\,v^2$ ☆

$+B^{-3}\,r\,v^2$ ☆

$-B^{-1}\,r\,v^2$ ☆ 11 ▲ 4

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Términos en $\mathcal{R}^3 \vec{v}$ que se convierten en términos en $\mathcal{R}^2 \vec{v}$.

$$+B^{-1}(r - \vec{V}\cdot\vec{r}) = B^{-1}\mathcal{R}^{-1} \qquad ①$$

$$+2B(\vec{V}\cdot\vec{v})(r - \vec{V}\cdot\vec{r}) = \cancel{2B\mathcal{R}^{-1}}\; 2B(\vec{V}\cdot\vec{v})\mathcal{R}^{-1} \qquad ②$$

$$-2B(\vec{V}\cdot\vec{v})^2(r - \vec{V}\cdot\vec{r}) = -\cancel{2B\mathcal{R}^{-1}} \qquad ③$$

$$-B^{-1}v^2(r - \vec{V}\cdot\vec{r}) = -B^{-1}v^2\mathcal{R}^{-1} \qquad ④$$

$$\boxed{\begin{array}{l} +B^{-1}\mathcal{R}^{-1} \\ +2B(\vec{V}\cdot\vec{v})\mathcal{R}^{-1} \\ -2B(\vec{V}\cdot\vec{v})^2\mathcal{R}^{-1} \\ -B^{-1}v^2\mathcal{R}^{-1} \end{array}}$$

Términos en $\mathcal{R}^2\vec{V}$ en el desarrollo de $\frac{\vec{a}}{M}$ en función de $\vec{r}$, de $\vec{V}$, de $\vec{A}$ y de $\vec{v}$.

$$+B(\vec{V}\cdot\vec{A})\,r\,b^{-2}$$
$$+2B^3(\vec{V}\cdot\vec{A})(\vec{r}\cdot\vec{v})$$
$$\cancel{-2B(\vec{V}\cdot\vec{v})}$$
$$-2B(\vec{A}\cdot\vec{v})\,r$$
$$-2B^3(\vec{V}\cdot\vec{A})(\vec{V}\cdot\vec{v})\,r$$
$$-2B(\vec{A}\cdot\vec{v})(\vec{r}\cdot\vec{v})$$
$$2B^3(\vec{V}\cdot\vec{A})(\vec{V}\cdot\vec{v})(\vec{r}\cdot\vec{v})$$
$$\cancel{+2B(\vec{V}\cdot\vec{v})^2}$$
$$+4B(\vec{V}\cdot\vec{v})(\vec{A}\cdot\vec{v})\,r$$
$$+2B^3(\vec{V}\cdot\vec{A})(\vec{V}\cdot\vec{v})^2\,r$$

$$+B^{-1}$$
$$\cancel{+2B(\vec{V}\cdot\vec{v})}$$
$$\cancel{-2B(\vec{V}\cdot\vec{v})^2}$$
$$-B^{-1}v^2$$

Términos en $\mathcal{R}^2 \vec{V}$ en el desarrollo de $\frac{\vec{\alpha}}{M}$ en función de $\vec{r}, \vec{V}, \vec{A}$ y $\vec{v}$.

$+B^{-1}b^{-2}$ +

$+B(\vec{V}\cdot\vec{A})\,r\,b^{-2}$ +

$+2B^3(\vec{V}\cdot\vec{A})(\vec{r}\cdot\vec{v})$ +

$-2B(\vec{A}\cdot\vec{v})\,r$ +

$-2B^3(\vec{V}\cdot\vec{A})(\vec{V}\cdot\vec{v})\,r$ +

$-2B(\vec{A}\cdot\vec{v})(\vec{r}\cdot\vec{v})$ +

$-2B^3(\vec{V}\cdot\vec{A})(\vec{V}\cdot\vec{v})(\vec{r}\cdot\vec{v})$ +

$+4B(\vec{V}\cdot\vec{v})(\vec{A}\cdot\vec{v})\,r$ +

$+2B^3(\vec{V}\cdot\vec{A})(\vec{V}\cdot\vec{v})^2\,r$ +

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Términos en $\mathcal{R}^3 \vec{v}$ en el desarrollo de $\frac{\vec{a}}{M}$ en función de $\vec{r}$, $\vec{V}$, $\vec{A}$ y $\vec{v}$.

$-B^{-1}(\vec{A}\cdot\vec{r})\, r\, b^{-2}$

$-B^{-3}\, r\, b^{-2}$

$+2B^{-1}(\vec{r}\cdot\vec{v})$

$+2B(\vec{A}\cdot\vec{r})(\vec{r}\cdot\vec{v})$

$-2B(\vec{V}\cdot\vec{v})(\vec{A}\cdot\vec{r})\, r$

$-2B^{-1}(\vec{V}\cdot\vec{v})\, r$

$-2B^{-1}(\vec{V}\cdot\vec{v})(\vec{r}\cdot\vec{v})$

$-2B(\vec{V}\cdot\vec{v})(\vec{A}\cdot\vec{r})(\vec{r}\cdot\vec{v})$

$+2B(\vec{V}\cdot\vec{v})^2(\vec{A}\cdot\vec{r})\, r$

$+2B^{-1}(\vec{V}\cdot\vec{v})^2\, r$

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